Designing Pension Plans According to Consumption-Savings Theory*

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Abstract

We derive optimal characteristics of contribution rates into defined-contribution pension plans based on consumption-savings theory. Contribution rates should increase with age and decrease with the balance-to-income ratio. Using registry data from Sweden, we show that on average, individuals save according to those principles. However, almost half of the population behaves hand-to-mouth and does not undo the mandated constant contribution rates. In a quantitative model, designing contribution rates to follow the principles implies a 1.8% welfare gain, and less dispersed replacement rates, while maintaining the same average replacement rate. Results are robust to various sources of model misspecification, including temptation preferences.

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1 Introduction

Developed countries are undertaking reforms that separate pension systems from the fiscal budget. A typical feature of such reforms is to rely more on defined-contribution (DC) pension plans to reduce the fiscal risks associated with the pension system. OECD (2017) reports that 32 out of 34 member countries have mandatory or quasi-mandatory second-pillar pension provision for workers. Fifteen member countries have DC pension plans. Notably, these DC pension plans feature mandatory contributions, which are a constant fraction of income. The shift to DC pension plans raises important questions about contributions that we argue have so far been largely understudied.

In this paper, we study optimal contribution rates to mandatory DC pension plans. We start by demonstrating that constant contribution rates out of income are at odds with basic principles of consumption-savings theory. Instead, optimal contribution rates should follow two fundamental principles: they should increase with the age of the investor and decrease with the asset balance-to-income ratio. Using Swedish registry-based data, we show that there is large heterogeneity in the extent to which individuals incorporate these principles in their savings decisions outside of the mandatory pension system. In particular, while individuals with sufficient liquid wealth save in line with the highlighted principles, almost half of the population behaves hand-to-mouth and therefore their total savings are exactly equal to the mandated, constant fraction of income. We thus build a rich quantitative life-cycle model, including a detailed pension system, and propose a more flexible design of contribution rates that adheres to the two fundamental principles. Based on the model, we propose a simple rule for contribution rates that significantly improves consumer welfare compared to an existing pension system with constant contribution rates, while maintaining the same average replacement rate. Moreover, the dispersion in replacement rates is substantially reduced compared to the dispersion under constant contribution rates. Finally, we show that the welfare benefits of redesigning contribution rates according to consumption-savings theory are robust to various sources of potential model misspecification, including potential problems of temptation and self control.

Our first contribution is that we formulate optimal principles for savings relative to income, i.e., savings rates. In a DC plan, participants accumulate an asset balance gradually over the course of their working life. This allows a close comparison and benchmarking of plans to standard consumption-savings theory. We make use of this feature and build a stylized three-period life-cycle model to highlight two guiding principles for optimal savings rates. First, optimal savings rates depend on expected income growth: the steeper the income profile, the lower the optimal savings rates for young workers. Plan participants with increasing income over their working life would thus optimally choose savings rates that increase with age. Second, optimal savings rates are a function of the asset balance-to-income ratio. The fundamental desire to smooth consumption between working life and retirement implies that savings rates should be lower if either current income falls or the asset balance increases, for instance due to high returns on previous savings. Constant mandatory contribution rates are at odds with both of these principles.

Our second contribution is to investigate to what extent individuals empirically adhere to these principles in their non-mandated savings outside of the pension system. Using Swedish registry-based data, we analyze a data sample that is representative of the Swedish population and contains comprehensive information about individuals' balance sheets, income and demographics. The panel dimension of the data together with information about each financial security held and its return enables us to measure individuals' savings rates into financial wealth. We show that on average, individuals' savings behavior is consistent with the predictions from the stylized model and the magnitudes are economically sizable. First, higher expected income growth is associated with lower savings rates and average savings rates increase with age. Second, we use regression analysis (both OLS and IV) to estimate the response of savings rates to shocks to the asset balance and to income. On average, individuals indeed decrease their savings rates after positive shocks to their asset balance-to-income ratio. Having said that, these average responses mask substantial heterogeneity in the cross-section. Almost half of the individuals behave as hand-to-mouth consumers and hold less than one month of income as liquid assets. While individuals with higher liquid assets behave in a way that is consistent with the principles, hand-to-mouth individuals do not save in liquid assets and therefore do not. This large group of individuals thus only saves in the form of mandated pension contributions, which are a constant fraction of income. They do not undo this rigid design.

Our third contribution is to study the benefits of designing a simple policy rule for mandatory contribution rates in a realistic economic environment. We build a quantitative, heterogeneous agents, life-cycle model that features risky labor income, a pension system with three pillars of retirement savings, and portfolio choice. We calibrate the model to the Swedish institutional setting—a setting that is often considered a model for other countries. Our proposal is very concrete: Starting from 1.31 percent of income, the contribution rate should increase by 0.3 percentage points for every year of age. Furthermore, investors who fall short by 1 percent from the target balance-to-income ratio for their age should increase their contribution rate by 0.15 percentage points.

Through its dependency on age, this rule provides both liquidity and consumption benefits for the first half of working life. For example, the average consumption of 30-year-old investors increases by 3.8 percent. Moreover, the dependency on the asset balance-to-income ratio lets contribution rates counteract shocks to income. Disposable income after pension contributions thereby becomes less volatile so that investors face less period-by-period risk in available cash-flows. Consequently, our proposed rule implies a substantial welfare gain. In terms of consumption equivalent variation, the gain is on average 1.8 percent. The insight from the three-period model hence holds in the fully calibrated life-cycle model: Designing contribution rates according to consumption-savings theory can improve welfare while maintaining the same average level of replacement rates.

An aspect that is at the core of the discussion of pension systems is the replacement rate: the ratio of resources during retirement relative to resources available during working life. While the level of replacement rates is important, there is increasing focus on inequality in replacement rates. For instance, Hagen, Laun, and Palme (2022) report that in Sweden there is considerable dispersion in total replacement rates, ranging from 85 percent at the tenth percentile to 120 percent at the 90th percentile. We show that in the calibrated life-cycle model, redesigning contribution rates according to consumption-savings theory substantially reduces the standard deviation in replacement rates of the DC account compared to the current system.

Finally, we demonstrate that our results are robust to various potential sources of model misspecification. In particular, there is a long history of literature that justifies the existence of mandatory pension or social security systems with the consideration that investors might have time-inconsistent preferences.[1] We generalize investor preferences to allow for temptation and self control as in Gul and Pesendorfer (2001, 2004) and re-calibrate the model for a wide range of possible degrees of self control. While the exact magnitude of the welfare benefits varies, allowing contribution rates to follow the basic principles turns out to be welfare improving for the whole range of considered parameter values. We also investigate robustness to misspecification in the other preference parameters. As with temptation and self control, the redesigned rule for contribution rates is welfare improving for a wide range of parameter values. Lastly, we compare the welfare benefits of redesigning contribution rates to the welfare effects of changing other margins of pension plans, namely the asset allocation and decumulation strategy. We find that redesigning contribution rates has welfare benefits that are an order of magnitude larger than the welfare consequences of altering these other aspects. This emphasizes the importance of focusing on contribution rates when optimizing pension systems.

Our analysis relates to three strands of the literature on pension plan design and savings rates of individuals and households. First, there is an ongoing debate about auto-enrollment into pension

[1] In a seminal work, Feldstein (1985) analyzed the optimal level of social security if households are myopic, i.e., if they have a shorter planning horizon than their life-span. İmrohoroğlu, İmrohoroğlu, and Joines (2003) conduct a similar analysis in an incomplete markets, heterogeneous agents setup where households have time-inconsistent preferences in the form of quasi-hyperbolic discounting as in Laibson (1997). Andersen and Bhattacharya (2021) show analytically in a three-period setup that both age-dependent and constant contribution rates into mandatory pension savings can be welfare improving compared to laissez-faire if individuals are sufficiently time-inconsistent.

plans and auto-escalation of contribution rates, in particular for the U.S., where DC pension plan designs vary more (see, e.g., Madrian and Shea (2001) and Choi, Laibson, Madrian, and Metrick (2003) for auto-enrollment and Thaler and Benartzi (2004) for auto-escalation). Our results suggest that designing defaults that involve auto-enrollment and automatic adjustments of the contributions could benefit from conditioning on individual circumstances, as our proposed rule suggests. Our finding that contributions rates optimally should increase with 0.3 percentage points per year is broadly consistent with the life-cycle profiles of contributions documented by Parker, Schoar, Cole, and Simester (2022).[2]

Second, there is strong concern in the literature that many consumers lack the necessary financial literacy to make informed retirement planning decisions (see Lusardi and Mitchell (2014) for an overview). As a mandatory DC pension plan, the design we propose relieves individuals from the majority of these complex questions. Moreover, in our proposed design, the mandatory contribution rates follow the principles of optimal consumption-savings theory. This allows financially illiterate households to get closer to the optimal retirement savings behavior. Third, our proposed rule features automatic adjustments due to income shocks. It is thus in line with, for instance, Beshears, Choi, Iwry, John, Laibson, and Madrian (2020), who discuss different designs of savings accounts that would enable individuals to build emergency savings and self-insurance against transitory income shocks.

Our analysis also relates to an ongoing policy debate fueled by the COVID-19 pandemic that concerns whether individuals should be able to withdraw balances from retirement accounts, such as 401(k). There are good arguments in favor of either side; on the one hand, if individuals are living hand-to-mouth, the welfare gain from allowing early withdrawals in emergency situations is large. On the other hand, individuals may miss out on high returns as financial markets reverse. Moreover, if investors are time inconsistent, they might withdraw excessively and thus end up in a situation with insufficient savings for retirement. Our proposal does not involve early withdrawals and thus avoids the associated perils. Put differently, our analysis centers attention to the cash-flow relief from automatic adjustments of contributions. We wish to highlight that the cash-flow reliefs associated with our proposed rule for the contribution rate attain more than one-third of the maximum welfare gain associated with a laissez-faire policy under the assumption of standard rational expectations, and up to one half of the maximum gain if individuals suffer from the temptation to overconsume. This suggests that allowing for pre-withdrawals, possibly against a penalty fee, at most implies an additional average gain of less than two-thirds or one half, respectively. Our findings thus imply that a flexible design of contribution rates substantially diminishes the value of

[2]The age-dependency of contribution rates also relates to the literature on optimal taxation which finds that in the presence of increasing life-cycle profiles of income and borrowing constraints taxes should be age-dependent (see, e.g., Hubbard and Judd (1986), İmrohoroğlu (1998), and Conesa, Kitao, and Krueger (2009))

outright pre-retirement withdrawals.

Relative to the existing literature, Larsen and Munk (2023) and Fischer, Jensen, and Koch (2022, 2023) are perhaps most similar to our study. These papers also study how to design optimal contribution rates given that pension plans are mandatory. Larsen and Munk (2023) investigate contribution rates that depend on age whereas we derive fundamental principles for consumption-savings theory and hence base our policy proposal on both age and the balance-to-income ratio. In addition, we show that empirically, a large share of the population behaves as hand-to-mouth consumers and hence is not able to follow optimal savings strategies in an environment with high mandatory contribution rates such as Sweden. Fischer, Jensen, and Koch (2023) derive a closed-form solution for age-dependent contribution rates in a deterministic environment. Fischer, Jensen, and Koch (2022) analyze the interaction of the design of contribution rates with home ownership. Pries (2007) uses a quantitative life-cycle model to study labor supply responses and welfare effects associated with a reform of U.S. Social Security to a system of individual accounts with age-dependent contribution rates.

The paper proceeds as follows. Section 2 sets up a stylized consumption-savings model to illustrate the basic principles of optimal contribution rates for a life-cycle investor. Section 3 showas that empirically, individuals on average save in line with these principles but that a large fraction of the population does not undo the mandated, constant contribution rates. Section 4 presents our quantitative life-cycle model, which incorporates a pension system that offers flexibility in contribution rates. Section 5 uses this model to design optimal rules for contribution rates that satisfy the principles of consumption-savings theory and presents their welfare implications. Section 6 demonstrates the robustness of the results to model misspecification and benchmarks the welfare benefits of redesigning contribution rates against the welfare effects of altering other margins of pension plans. Section 7 concludes.

2 Consumption-savings theory and optimal savings rates

In this section, we consider a stylized life-cycle consumption-savings model that highlights the determinants of the optimal savings rate.[3] We derive specific characteristics of optimal savings behavior and discuss their implications for the design of optimal pension plans.

[3] We use the term *savings rate* since the stylized model abstracts from the pension system. In our full model, we use the term *contribution rate*, which is the savings rate that a defined-contribution pension plan stipulates.

2.1 A stylized life-cycle model

The life cycle consists of three distinct periods: young working life, middle-aged working life, and retirement. Investors receive exogenous, deterministic income in the two working-life periods but do not receive any income in the retirement period. They optimally choose savings—and hence savings rates out of current income—in the working-life periods to smooth consumption over the life cycle.

An investor i is born in period $t = 1$ with no assets and with an exogenous income profile $Y_{i,1}$ and $Y_{i,2}$ in periods $t = 1$ and $t = 2$, respectively. Savings bear risk-free interest R_1 and R_2 in the two periods. In the retirement period ($t = 3$), investors do not receive any income and thus simply consume the savings that they accumulated from previous periods. Individuals maximize discounted lifetime utility where they discount future periods with discount factor β and have logarithmic flow utility. In detail, the investors choose a sequence of consumption $\{C_{i,t}\}_{t=1}^{3}$ and of savings $\{A_{i,t}\}_{t=1}^{2}$ to solve the following optimization problem:

$$\max_{\{C_{i,t}\}_{t=1}^{3}, \{A_{i,t}\}_{t=1}^{2}} \quad \log(C_{i,1}) + \beta \log(C_{i,2}) + \beta^2 \log(C_{i,3}) \tag{1}$$

$$\text{s.t.} \quad C_{i,1} = Y_{i,1} - A_{i,1} \tag{2}$$

$$C_{i,2} = A_{i,1} R_1 + Y_{i,2} - A_{i,2} \tag{3}$$

$$C_{i,3} = A_{i,2} R_2. \tag{4}$$

Pension plans are described by their savings rates out of current income. We therefore define savings rates in working life as

$$\lambda_{i,1} = \frac{A_{i,1}}{Y_{i,1}} \tag{5}$$

$$\lambda_{i,2} = \frac{A_{i,2} - A_{i,1} R_1}{Y_{i,2}}. \tag{6}$$

Note that in $t = 2$, the savings rate measures the additional savings over and above savings brought into the period, as opposed to total savings $A_{i,2}$.

2.2 Characteristics of optimal savings rates

The following propositions characterize the optimal savings rate for young and middle-aged individuals. The proofs of all propositions can be found in Appendix A.

Proposition 1 *The optimal savings rate in t=1 decreases in expected income growth.*

Explicitly, the optimal savings rate in $t = 1$ is equal to

$$\lambda_1^* = \frac{(\beta + \beta^2)}{(1 + \beta + \beta^2)} - \frac{1}{(1 + \beta + \beta^2)} \frac{1}{R_1} \frac{Y_{i,2}}{Y_{i,1}}. \tag{7}$$

The intuition for this proposition is that, as income later in working life increases, the individual optimally wants to delay savings for retirement. If income early in working life is sufficiently small compared to later in working life, i.e., income growth is sufficiently large, the individual optimally would like to borrow when she is young.

Proposition 2 *The optimal savings rate in t=2 decreases in the asset balance-to-income ratio.*

Explicitly, the optimal savings rate in $t = 2$ is equal to

$$\lambda_2^* = \frac{\beta}{1 + \beta} - \frac{1}{1 + \beta} \frac{A_{i,1} R_1}{Y_{i,2}}. \tag{8}$$

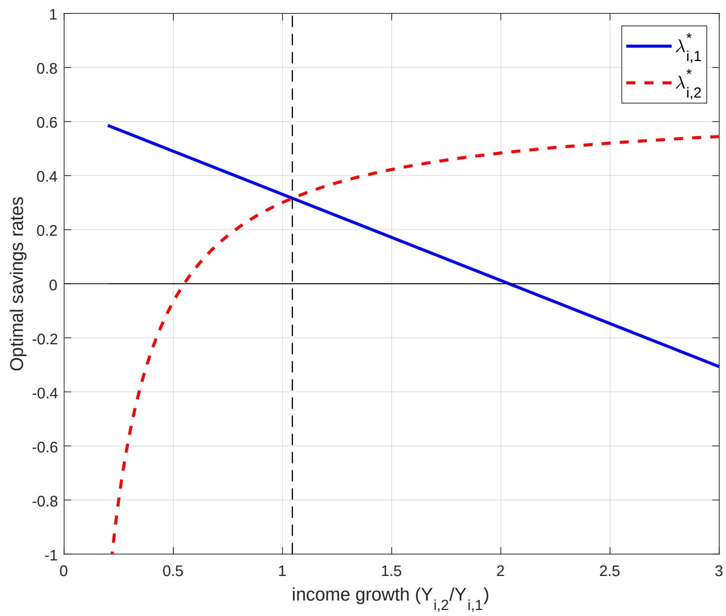
The intuition for this proposition is that individuals want to smooth their consumption between working life and retirement. If they have already accumulated high savings relative to their current income, they optimally do not want to save much in addition to the savings that they have already accumulated. Moreover, if the balance of their savings is sufficiently large relative to their current income, the individual would optimally like to access these savings for current consumption, leading to a negative savings rate out of current income.

Proposition 3 *There exists a specific income profile* $\frac{Y_{i,2}}{Y_{i,1}} = \kappa(R_1, \beta)$ *such that the optimal savings rates in* $t = 1$ *and* $t = 2$ *are exactly identical. For any other income profile, constant savings rates throughout working life are suboptimal.*

Figure 1 illustrates this proposition graphically for a particular combination of discount rates and interest rates. It depicts the optimal savings rates in period $t = 1$ and $t = 2$ as a function of income growth. The knife-edge case where the savings rates are optimally constant throughout working life is indicated with the vertical dashed line. The figure emphasizes that, for an income growth larger than κ, the individual would optimally like to postpone saving for retirement and hence would optimally choose to have a larger savings rate late in working life relative to earlier in working life.

Note that the threshold income growth κ is a function of the discount factor and interest rates. The model assumes for simplicity that all investors face the same interest rates. In reality, returns are likely to be heterogeneous due to both idiosyncratic differences in investment possibilities and common risk factors. In this case, the threshold level of income growth κ would be heterogeneous

Figure 1: Optimal savings rates in the three-period model



Note: The figure illustrates the optimal savings rates in periods $t = 1$ and $t = 2$ as a function of income growth $\frac{Y_{i,2}}{Y_{i,1}}$. The discount factor and return in period 1 are set to $\beta = 0.95$ and $R_1 = 1.1$, respectively. The dashed vertical line indicates the value $\frac{Y_{i,2}}{Y_{i,1}} = \kappa(R_1, \beta)$, where optimal savings rates are exactly constant throughout working life.

across investors, and the distribution of thresholds would vary over time with the risk factors. Thus, a constant savings rate for all investors and across all time periods would not be optimal.

2.3 Implications for pension plan design

Existing mandatory pension plans typically feature constant savings rates (contribution rates) out of income across all age groups and irrespective of individual circumstances. We have shown that such a design is at odds with the basic principles of optimal consumption-savings behavior. In particular, we have shown two deficiencies of constant savings rates. First, young individuals, who typically expect their income to grow substantially, would like to postpone saving for retirement. They would thus choose to have an increasing age profile of savings rates. Second, we have shown that optimal savings rates later in working life vary with the asset balance-to-income ratio. The fundamental desire to smooth consumption over working life and retirement implies that workers should optimally reduce their current savings rates if their accumulated asset balance is larger relative to current income. This could either be because they had high income and thus high contributions in the past, because they received a particularly good return on their earlier savings, or because their current income is comparatively low. We thus conclude from this section that an optimal design of pension plans should both feature an increasing age profile of savings rates and allow savings rates to adjust with the asset balance-to-income ratio. We will use these insights from the stylized consumption-savings model when we design an optimal pension plan in the quantitative model.

3 Empirical tests of the principles for consumption and savings

Do investors empirically behave in a way that is consistent with these fundamental principles? We use Swedish registry-based data to investigate to what extent individuals' consumption-savings behavior is consistent with the principles we derived in the three-period model. We focus on the savings in liquid, non-mandated savings as the mandatory pension contributions do not allow for any flexibility. Importantly, the pension system in Sweden—an institutional setting that is often a model for other countries—features large mandatory pension contributions of in total 23 percent of income (for the first and second pillar combined). This implies that the liquid, non-mandated savings that individuals can control are assets that are saved in addition to those already considerable contributions.

3.1 Data

Our dataset is a representative sample of the Swedish population for 2000–2007.[4] It contains comprehensive information about individuals' balance sheets, income and demographics. Through the tax and financial registries, we are able to observe stocks, cash in bank accounts, mutual funds, bonds and various types of financial securities held by individuals. This is possible because a wealth tax was levied on individuals during our sample period, which by law required individuals to disclose their assets, earnings and income to the tax authority. Furthermore, standard sociodemographic variables, such as age and income, are observable. See, e.g., Calvet, Campbell, and Sodini (2007), Dahlquist, Setty, and Vestman (2018), and Di Maggio, Kermani, and Majlesi (2020) for details.

The detailed information in the data combined with the longitudinal dimension enables us to measure both individuals' asset balance in financial wealth and their savings rates in financial wealth. The data challenge in constructing the savings rates lies in the need of longitudinal data

[4]We start from a population data set of all Swedes for 2000–2007 and then employ a sequence of sample restrictions that improve precision in measurement. See Table A.1 in the Appendix.

and detailed asset returns. To be precise, the balance in financial wealth can be constructed as:

$$A_{it} = \sum_{k=1}^{K} q_{itk} \cdot P_{kt}, \tag{9}$$

where K is the number of financial securities in the portfolio, q_{itk} indicates the number of shares held of security k—a stock, mutual fund or bond—by individual i at the end of year t, and P_{kt} is the price per share. In addition, we measure the balances of bank accounts and capital insurance accounts. Based on this information, we can also compute the gross return:

$$R_{it}^{A} = \sum_{k=1}^{K} w_{ikt-1} \frac{P_{kt} + D_{kt}}{P_{kt-1}}, \tag{10}$$

where w_{ikt-1} is the portfolio weight for individual i in asset k at time $t-1$ and P_{kt} and D_{kt} are the end-of-year price and financial payouts (e.g., dividends) by asset k at time t, respectively.[5]

We construct a cash-flow measure of savings in liquid financial wealth as follows:

$$\Delta\widetilde{A}_{it} = A_{it} - A_{it-1} \times R_{it}^{A}, \tag{11}$$

and from this flow we construct the savings rate

$$\widetilde{\lambda}_{it} = \frac{\Delta\widetilde{A}_{it}}{Y_{it}^{\text{Disp}}}, \tag{12}$$

where Y_{it}^{Disp} is disposable income (gross labor income minus taxes plus transfers). The variable $\widetilde{\lambda}_{it}$ is the empirical counterpart to the savings rates in Section 2.[6] This is a cash-flow based measure of the savings rate that resembles the ones constructed by e.g., Fagereng, Blomhoff Holm, Moll, and Natvik (2019) and Bach, Calvet, and Sodini (2018). This is sometimes referred to as the active savings rate since it reflects active decisions to withdraw or contribute during year t from the end-

[5]The gross return for bonds and bank accounts is set to 1, while the gross return for stocks and mutual funds with missing return data is set to the return of the MSCI World Gross Index. The financial securities held in the so-called *capital insurance accounts* are unobservable. Therefore, we set the gross return equal to the return of the all-share Stockholm Stock Exchange, in particular the SIX Return Index.

[6]To be clear, Statistics Sweden also reports asset balances for each asset class. Our way of computing the cash-flow measure of savings, (11), gives almost identical values as an approach that uses Statistics Sweden's precomputed asset balances; that is, $\Delta\widetilde{A}_{it} = A_{it} - A_{it-1} \times R_{it}^{A} \approx \Delta b_{it} + \Delta v_{it} + \Delta\psi_{it} - y_{it}^{v}$, where Δb_{it} is the change in bank account balances, Δv_{it} is the active rebalancing of stocks, mutual funds, and bonds, $\Delta\psi_{it}$ is the net-change in capital insurance accounts, and y_{it}^{v} is after-tax financial income, particularly dividends from stocks, interest payments from bank accounts, and coupons from bonds. The correlation between the left-hand side and the measure after the approximation is 0.98. The difference between the two measures has a mean and median of 356 and 0 SEK, respectively.

Table 1: Summary statistics

	Full sample		Stock market participants		Non-participants	
	Mean	Median	Mean	Median	Mean	Median
Age	43.441	43.000	44.425	44.000	42.181	41.000
Disposable income (Y_{it}^{Disp})	193.873	184.140	203.198	192.120	181.934	174.952
Liquid financial wealth (A_{it})	95.722	19.546	153.204	62.639	22.132	0.000
Net worth	247.485	75.225	391.046	213.902	63.699	-4.769
Assets-to-income ($A_{it}/Y_{it}^{\text{Disp}}$)	0.380	0.104	0.593	0.322	0.106	0.000
Hand-to-mouth	0.472	0.000	0.239	0.000	0.771	1.000
Stock market participation	0.561	1.000	1.000	1.000	0.000	0.000
Savings rate ($\widetilde{\lambda}_{it}$)	0.024	0.000	0.030	0.006	0.016	0.000
Observations	16,134,639	16,134,639	9,058,653	9,058,653	7,075,986	7,075,986

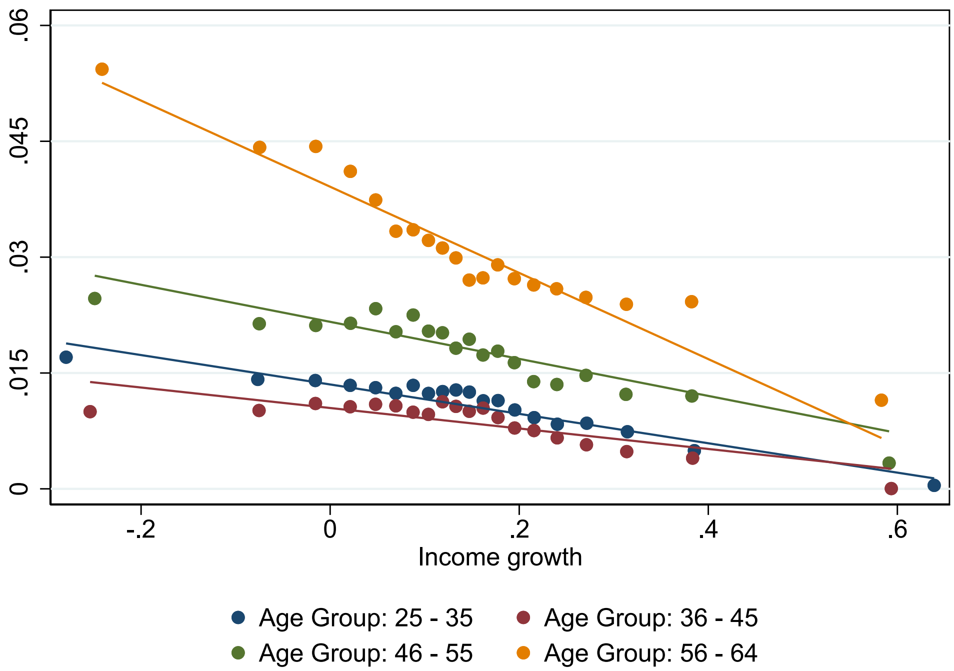
Note: Net worth, liquid financial wealth and disposable income in SEK 1,000. Stock market participation is a dummy variable equal to 1 if the individual holds stocks directly or equity mutual funds outside the pension system. Assets-to-income is short for liquid financial wealth divided by disposable income. Hand-to-mouth is a dummy variable equal to 1 if assets-to-income is less than or equal to 1/12. The hand-to-mouth individuals represent 47.2% of the entire sample, with an average of 52.2% for years 2002-2005 and 37.0% for 2006-2007 (due to changes in bank account reporting requirements).

of-year $t-1$ asset balance, A_{it-1}. This makes $\widetilde{\lambda}_{it}$ a measure of the savings rate that is analogue to pension plans' stipulated contribution rates.

Table 1 reports means and medians in our sample for the most important variables in our analysis. The average age is 43 year, that is mid-working life. Annual disposable income is SEK 193,873, liquid financial wealth is SEK 95,722.[7] The mean assets-to-income ratio is 0.380. The distribution is however skewed – the median is only 0.104. Importantly, a lot of individuals in our sample only have about one month's worth of disposable income in liquid financial wealth to use as a savings buffer. Consistent with Kaplan, Violante, and Weidner (2014), we define an individual as being hand-to-mouth if the assets-to-income ratio is less than 1/12—47.2 percent of individuals are hand-to-mouth according to this definition. Despite the low asset balances, 56.1 percent of the individuals are stock market participants in the sense that they own either stocks or equity mutual funds directly, outside of their DC pension plans. The average savings rate is 2.4 percent, but the median is zero. Part of our empirical analysis relies on risky idiosyncratic returns so we also report statistics for stock market participants and non-participants. Participants on average earn more and have higher liquid financial wealth. Only 23.9 percent of stock market participants are hand-to-

[7] In 2022, the SEK/USD exchange rate was around 10. During our sample period, it has fluctuated between 6 and 10. We henceforth report numbers in SEK.

Figure 2: Income growth, age and savings rates



Note: The figure is a binned scatter plot of savings rates against income growth rates. The savings rate is the average over two years, $(\widetilde{\lambda}_{i,2002} + \widetilde{\lambda}_{i,2003})/2$. Income growth is defined as $y^{\text{Disp}}_{i,late} - y^{\text{Disp}}_{i,early}$, where $y^{\text{Disp}}_{i,early} = \log((Y^{\text{Disp}}_{i,2002} + Y^{\text{Disp}}_{i,2003})/2)$ and $y^{\text{Disp}}_{i,late} = \log((Y^{\text{Disp}}_{i,2006} + Y^{\text{Disp}}_{i,2007})/2)$.

mouth whereas 77.1 percent of non-participants are. This is consistent with their savings rates: 3.0 percent for participants compared to 1.6 percent for non-participants.

3.2 The role of age and expected income growth

Our three-period model predicts that savings rates should decrease in expected income growth (Proposition 1). For most individuals, this implies that savings rates should increase with age. We investigate these relationships based on the following regression:

$$\widetilde{\lambda}_{it} = \beta_0 + \beta_1 \times (y^{Disp}_{i,t+s} - y^{Disp}_{i,t}) + \varepsilon_{it}, \tag{13}$$

where $y^{Disp}_{i,t+s} - y^{Disp}_{i,t}$ is the log difference in disposable income between year $t+s$ and t and ε_{it} is an error term. The coefficient of interest is β_1, which is the elasticity of the savings rate with respect to income growth.

Figure 2 reports a binned scatter plot of this regression, where income growth has been interacted with dummy variables indicating the age group of the individual, and income growth is

Table 2: Savings rates and the role of expected income growth and age

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
		Full Sample		Hand-to-Mouth		Non Hand-to-Mouth	
$y^{Disp}_{i,t+4} - y^{Disp}_{i,t}$	-0.026***	-0.056***		-0.005***		-0.037***	
	(0.001)	(0.003)		(0.000)		(0.001)	
$\mathbb{1}$[Age 25 – 35]		-0.026***	-0.025***		0.006***		-0.019***
		(0.001)	(0.000)		(0.000)		(0.000)
$\mathbb{1}$[Age 36 – 45]		-0.029***	-0.028***		0.006***		-0.025***
		(0.001)	(0.000)		(0.000)		(0.000)
$\mathbb{1}$[Age 46 – 55]		-0.017***	-0.017***		0.004***		-0.014***
		(0.001)	(0.000)		(0.000)		(0.001)
$(y^{Disp}_{i,t+4} - y^{Disp}_{i,t})\times$ $\mathbb{1}$[Age 25 – 35]		0.037*** (0.003)					
$(y^{Disp}_{i,t+4} - y^{Disp}_{i,t})\times$ $\mathbb{1}$[Age 36 – 45]		0.043*** (0.003)					
$(y^{Disp}_{i,t+4} - y^{Disp}_{i,t})\times$ $\mathbb{1}$[Age 46 – 55]		0.032*** (0.003)					
Constant	0.019***	0.039***	0.038***	-0.010***	-0.016***	0.048***	0.063***
	(0.000)	(0.001)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
F $\mathbb{1}$[Age 25 – 35] = $\mathbb{1}$[Age 36 – 45]		143***	161***		4**		245***
F $\mathbb{1}$[Age 36 – 45] = $\mathbb{1}$[Age 46 – 55]		1,310***	2,400		317***		759***
F $\mathbb{1}$[Age 46 – 55] = $\mathbb{1}$[Age 56 – 64]		935***	2,645***		364***		721***
Observations	1,602,790	1,602,790	2,386,418	847,229	1,1907,47	755,561	1,195,671
R^2	0.002	0.006	0.006	0.000	0.002	0.002	0.003

Note: The table reports estimates of regressing savings rates on income growth rates and age dummy variables using OLS. The savings rate is the average over two years, $(\widetilde{\lambda}_{i,2002} + \widetilde{\lambda}_{i,2003})/2$ and income growth is defined as $y^{\text{Disp}}_{i,t+4} - y^{\text{Disp}}_{i,t}$, where $y^{\text{Disp}}_{i,t} = \log((Y^{\text{Disp}}_{i,2002} + Y^{\text{Disp}}_{i,2003})/2)$ and $y^{\text{Disp}}_{i,t+4} = \log((Y^{\text{Disp}}_{i,2006} + Y^{\text{Disp}}_{i,2007})/2)$. Columns (1)–(3) report estimates for the entire sample. Columns (4)–(5) report estimates for hand-to-mouth individuals. Columns (6)–(7) report estimates for non hand-to-mouth individuals. $* = p < 0.10$, $** = p < 0.05$, $*** = p < 0.01$.

measured four years ahead of the observed savings rate ($s = 4$). If income is at least partially predictable, then $y^{Disp}_{i,t+4} - y^{Disp}_{i,t}$ is a proxy for expected income growth at t. The interaction with age groups is intended to isolate the effect of expected income growth since it varies over the life cycle. For all age groups, there is a strong negative relationship between income growth in the coming four years and savings rates today. Further, individuals who are older than 45 years save more than younger ones. Table 2 reports detailed estimates. Column (1) shows that there is a strong negative relationship between income growth and savings rates. A ten-percent increase in income over the next four years is on average associated with 0.26 percentage points lower savings rates today, compared to individuals who except no income growth.

Column (2) shows variation in response to income growth over the life-cycle. While individuals aged 56–64 decrease savings the most with higher income growth, 0.56 percent points for a ten-percent increase in income, individuals aged 26–55 lower savings rates by 0.13 to 0.24 percentage points. The negative relationship between expected income growth and savings rates is consistent with Proposition 1. Additionally, Column (3) illustrates that younger individuals (25–45 years) save on average 2.5 to 2.8 percentage points less compared to older individuals. This is consistent with Proposition 3.

So far, we showed that individuals on average adjust their savings rates in liquid financial wealth based on age and expected income growth. However, this masks substantial heterogeneity across investors. Columns (4)–(7) of Table 2 present the estimation results separately for hand-to-mouth and non hand-to-mouth investors. Hand-to-mouth individuals display almost no variation in savings rates depending on future income growth and age. Compared to the full sample, estimates are only one fifth as large (Column (4)), and there is no meaningful variation in savings rates over the life-cycle (Column (5)). The last two columns illustrate that essentially all the adjustments in the average savings rates are originating from the non hand-to-mouth individuals, who have more liquid financial wealth to begin with, and by definition have greater ability to dissave, if deemed appropriate. We conclude that only half of the individuals adjust their savings rates to future income growth in a meaningful way.

3.3 The role of shocks to the asset balance-to-income ratio

Our three-period model predicts that the optimal savings rate decreases in the asset balance-to-income ratio (Proposition 2). We investigate if this is supported empirically by analyzing the savings responses to shocks to the two components—asset balance and income—separately.

A positive shock to the asset balance ceteris paribus increases the asset balance-to-income ratio. Theory predicts that this should lead to a reduction in the savings rate. We test this by analyzing the response of savings rates to idiosyncratic shocks to returns. To do so, we estimate the following

Table 3: Savings rates and the role of asset-balance shocks

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
	OLS		IV				
$\frac{A_{it-1}}{Y_{it}^{Disp}} \times R_{it}^{A}$	-0.106***	-0.103***	-0.104***	-0.154***	-0.122***	-0.138***	-0.088***
	(0.006)	(0.006)	(0.006)	(0.014)	(0.008)	(0.017)	(0.008)
Age Group	25-64	25-64	25-64	25-35	36-45	46-55	56-64
Controls	No	Yes	Yes	Yes	Yes	Yes	Yes
Individual FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Year FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes
First-stage *F*			74,704***	49,735***	32,074***	93,935***	17,881***
R^2	0.318	0.319					
Observations	8,900,223	8,872,650	8,872,650	2,212,862	2,282,590	2,178,616	1,820,876

Note: This table reports estimates of regressing savings rates on an interaction between asset balance-to-income ratios and asset returns. The sample is restricted to stock market participants (i.e., individuals with non-zero stock, mutual fund or capital insurance holdings). The control variables are Y_{it}^{Disp}, NW_{it-1} and $ND_{it,t-1}$. Standard errors, clustered at the level of the individual and the individual's largest security holding, are in parentheses. The individual's largest security holding is the particular stock or mutual fund—identified by their International Securities Identification Number (ISIN)—with the largest weight in the individual's financial asset portfolio. When the largest holding is bonds, bank accounts or capital insurance accounts, we classify the largest security holding by its respective asset type. Singleton groups are excluded. Table A.2 reports first-stage IV estimates. $* = p < 0.10$, $** = p < 0.05$, $*** = p < 0.01$.

regression using OLS and IV, as in Di Maggio, Kermani, and Majlesi (2020):

$$\widetilde{\lambda}_{it} = \theta_i + \delta_t + \beta_1 \frac{A_{it-1}}{Y_{it}^{\text{Disp}}} \times R_{it}^{A} + \beta_2 Y_{it}^{\text{Disp}} + \beta_3 NW_{it-1} + \beta_4 ND_{it,t-1} + \varepsilon_{it}, \tag{14}$$

where θ_i and δ_t are individual and year fixed effects, respectively, and ε_{it} is an error term. The control variables are disposable income Y_{it}^{Disp}, one-year lagged net worth NW_{it-1}, and a dummy variable $ND_{it,t-1}$ equal to 1 if the individual did not receive a dividend at time t or at time $t-1$. The variable A_{it-1} is the end-of-year $t-1$ (beginning-of-year t) asset balance, and R_{it}^{A} is the gross return during year t of the portfolio, had it been unchanged throughout the year. The coefficient of interest is β_1, which measures the response in the savings rate upon a change in the asset balance-to-income ratio, originating from an idiosyncratic return shock. To have meaningful variation in R_{it}^{A} we restrict the sample to stock market participants. According to Proposition 2, we expect $\beta_1 < 0$.

OLS estimates are reported in columns (1)–(2) of Table 3. The estimates indicate that an increase in the asset balance by one year's worth of disposable income reduces individuals' savings rates by 10.3 percentage points (holding disposable income constant). This is a strong response since it implies that the response in savings rates to such a change is of the same order of magnitude as typical pension plans' mandated contribution rates.

The remainder of the table presents IV estimates to address potential endogeneity, as in Di Maggio, Kermani, and Majlesi (2020). If individuals simultaneously adjust their portfolios and savings rates during the year, due to for instance macroeconomic news, then OLS estimates are biased. We therefore instrument $\frac{A_{it-1}}{Y_{it}^{\text{Disp}}} \times R_{it}^{A}$ with $\frac{A_{it-1}}{Y_{it}^{\text{Disp}}} \times \overline{R_{it}^{A}}$, where $\overline{R_{it}^{A}}$ is the individual's return on financial assets given the portfolio held at $t-2$.[8]

Our IV estimates are of similar magnitude to the OLS estimates: In the full sample, column (3), an increase in the asset balance by one year's worth of disposable income reduces savings rates by 10.4 percentage points. We also consider heterogeneity in savings responses for individuals of different age. The magnitude of the response in the savings rate decreases with age. The response of individuals who are 26–35 years old is fifty percent stronger than the average (-0.154; column (4)), and the response of individuals who are 56–64 years old is less strong (-0.088; column (7)). This is consistent with young individuals having higher marginal utility of consumption, for instance because of binding borrowing constraints.[9]

The asset balance-to-income ratio can also change due to shocks to its denominator—income. A negative shock to income ceteris paribus increases the asset balance-to-income ratio. Proposition 2 hence predicts that individuals should decrease their savings rate in response to adverse shocks to income. We test this by estimating the following regression:

$$\widetilde{\lambda}_{it} = \theta_i + \delta_t + \beta_1 \times (y_{i,t}^{Disp} - y_{i,t-1}^{Disp}) + \varepsilon_{it}, \tag{15}$$

where $y_{i,t}^{Disp} - y_{i,t-1}^{Disp}$ is the log difference in disposable income between year t and $t-1$, θ_i and δ_t are individual and time fixed effects, respectively, and ε_{it} is an error term. The coefficient of interest is β_1, which is the elasticity of the savings rate with respect to an income change.

Table 4 reports OLS and IV estimates. If individuals simultaneously adjust labor supply and savings rates then the OLS estimate suffers from endogeneity, as discussed in Fagereng, Guiso, and Pistaferri (2018). We therefore adopt their estimation strategy and use IV estimation, where we instrument individual i's income growth with the aggregate wage growth of individual i's employer. Column (1) reports the OLS estimate for the full sample. The response of individuals is small but qualitatively consistent with the prediction of our theoretical model: A ten-percent increase in

[8]Formally, $\overline{R_{it}^{A}} = \sum_{k=1}^{K} w_{ikt-2} \frac{P_{kt} + D_{kt}}{P_{kt-1}}$.

[9]Table A.3 considers an alternative regression specification that measures the response in $\Delta\widetilde{A}_{it}$ to changes in $A_{it-1} \times R_{it}^{A}$. The estimates are qualitatively the same.

Table 4: Savings rates and the role of income shocks

	(1)	(2)	(3)	(4)	(5)	(6)
	Full Sample		Hand-to-Mouth		Non Hand to Mouth	
	OLS	IV	OLS	IV	OLS	IV
$y_{it}^{Disp} - y_{it-1}^{Disp}$	0.022***	0.024***	0.009***	-0.004	0.046***	0.061***
	(0.000)	(0.008)	(0.000)	(0.002)	(0.001)	(0.017)
Individual FE	Yes	Yes	Yes	Yes	Yes	Yes
Year FE	Yes	Yes	Yes	Yes	Yes	Yes
Individual Clusters	Yes	Yes	Yes	Yes	Yes	Yes
Employer Clusters	No	Yes	No	Yes	No	Yes
First-stage *F*		6,950***		5,908***		3,358***
R^2	0.256		0.392		0.296	
Observations	16,134,639	12,121,447	7,318,708	5,626,553	8,060,550	6,785,791

Notes: The table reports estimates of regressing savings rates on changes in income by OLS and IV. Columns (1)–(2) report estimates for the entire sample. Columns (3)–(4) report estimates for hand-to-mouth individuals. Columns (5)–(6) report estimates for non hand-to-mouth individuals. The IV estimates in columns (2), (4) and (6) use the wage-bill growth of the employer as an instrument for the individual's income growth. Singleton groups are excluded. Table A.5 reports the first-stage estimates. $* = p < 0.10$, $** = p < 0.05$, $*** = p < 0.01$

income from $t-1$ to t increases the savings rate by 0.22 percentage points. Column (2) reports the corresponding IV estimate which is of similar magnitude: A positive income shock of ten percent increases the savings rate by 0.24 percentage points.

As before, these average results for the full sample mask substantial heterogeneity. Columns (3)–(4) show that for hand-to-mouth individuals, income shocks render much smaller or even no adjustments of savings rates. In contrast, individuals with more financial wealth respond two to three times more than what the full sample estimates suggest (Columns (5)–(6)).

We conclude that individuals differ vastly in how much their choices in liquid assets align with the fundamental principles of consumption-savings theory. Some individuals, with more liquid financial wealth, behave in line with the principles highlighted by the three-period model. Others, who live hand-to-mouth, naturally do not. In terms of total savings, this large group of hand-to-mouth individuals hence saves a constant, mandated fraction of income. We will now set up a quantitative life-cycle model to gauge the benefits of redesigning these mandatory contribution rates according consumption-savings theory.

4 A quantitative life-cycle model

We set up a life-cycle model featuring a detailed pension system as currently mandated in Sweden. The setup is an extension of Dahlquist, Setty, and Vestman (2018), which in turn builds on Viceira (2001), Cocco et al. (2005), and Gomes and Michaelides (2005).[10] It includes risky labor income, a consumption-savings choice, and a portfolio choice. We augment the model with a pension system in which individuals save in two pension accounts, from which they receive their pension as annuities. One of the accounts belongs to the first pillar of the pension system and is pay-as-you-go but with an individual notional balance. The other account is a standard defined-contribution pension account that represents the second pillar of the pension system. The focus of our analysis is to design optimal contributions into this account.

4.1 Model setup

Next, we describe the model's building blocks.

Demographics

We follow individuals from age 25 until the end of their lives.[11] The end of life occurs at the latest at age 100 but could occur before since individuals face an age-specific survival rate, ϕ_t. The life cycle is split into a working, or accumulation, phase and a retirement phase. From the age of 25 to 64, individuals work and receive labor income exogenously. They retire at 65.

Preferences

Individuals have Epstein and Zin (1989) preferences over a single consumption good. At age t, each individual maximizes the following:

$$U_t = \left(C_t^{1-\rho} + \beta \phi_t E_t \left[U_{t+1}^{1-\gamma} \right]^{\frac{1-\rho}{1-\gamma}} \right)^{\frac{1}{1-\rho}}, \quad (16)$$

$$U_T = C_T, \quad (17)$$

where β is the discount factor, $1/\rho$ is the elasticity of intertemporal substitution, γ is the coefficient of relative risk aversion, and $t = 25, 26, ..., T$ with $T = 100$. For notational convenience, we define

[10] Our model relates to Gomes et al. (2009), who consider portfolio choice in the presence of tax-deferred retirement accounts, and to Campanale et al. (2014), who consider a model in which stocks are subject to transaction costs, making them less liquid.

[11] We choose age 25 as the start of the working phase since Swedish workers do not fully qualify for occupational pension plans before that age.

the operator $\mathcal{R}_t(U_{t+1}) \equiv E_t\left[U_{t+1}^{1-\gamma}\right]^{\frac{1}{1-\gamma}}$.

Labor income

Let Y_{it} denote the labor income of an employed individual i at age t. During the working phase (up to age 64), the individual faces a labor income process with a life-cycle trend and persistent income shocks:

$$\begin{aligned} y_{it} &= g_t + z_{it}, && (18) \\ z_{it} &= z_{it-1} + \eta_{it} + \theta\varepsilon_t, && (19) \end{aligned}$$

where $y_{it} = \ln(Y_{it})$. The first component, g_t, is a hump-shaped life-cycle trend. The second component, z_{it}, is a permanent labor income component. It has an idiosyncratic shock, η_{it}, which is distributed $\mathrm{N}\big(-\sigma_\eta^2/2, \sigma_\eta^2\big)$, and an aggregate shock, ε_t, which is distributed $\mathrm{N}(-\sigma_\varepsilon^2/2, \sigma_\varepsilon^2)$. The aggregate shock also affects the stock return, and θ determines the contemporaneous correlation between the labor income and the stock return. We allow for heterogeneity in income as early as age 25 by letting the initial persistent shock, z_{i25}, be distributed $\mathrm{N}(-\sigma_z^2/2, \sigma_z^2)$.

During the retirement phase (from age 65 onwards), the individual has no labor income.[12] Pension is often modeled as a deterministic replacement rate relative to the labor income just before retirement.[13] However, in our model, the replacement rate is endogenously determined. The individual relies entirely on annuity payments from the savings accounts. Later, we discuss these accounts in detail.

Asset returns

The gross return on the stock market, R_{t+1}, develops according to the following log-normal process:

$$\ln(R_{t+1}) = \ln(R_f) + \mu + \varepsilon_{t+1}, \quad (20)$$

where R_f is the gross return on a risk-free bond and μ is the equity premium. Recall that the shock, ε_t, is distributed $\mathrm{N}(-\sigma_\varepsilon^2/2, \sigma_\varepsilon^2)$, so $E_t(R_{t+1} - R_f) = \mu$. Also recall that ε_t affects labor income in (19) and the correlation between stock returns and labor income is governed by the parameter θ.

[12] Hence, the retirement decision is not endogenous as in French and Jones (2011). More generally, we do not consider endogenous labor supply decisions as in Bodie et al. (1992) and Gomes et al. (2008).

[13] One exception is that of Cocco and Lopes (2011), who model the preferred DB or DC pension plan for different investors.

Three accounts for retirement savings

An individual has three financial savings accounts: (i) a notional account belonging to the pension system, (ii) a fully-funded DC account in the pension system, and (iii) a liquid account outside the pension system (which we simply refer to as financial wealth).

The first pillar of the pension system is a notional account. It provides the basis for the pension, is income based and evolves at the rate of the risk-free bond. Specifically, during the working phase, its balance evolves as follows:

$$N_{it+1} = (N_{it} + \lambda^{\mathrm{N}} Y_{it}) R_f, \tag{21}$$

where λ^{N} is the contribution rate for the notional account. As in Dahlquist, Setty, and Vestman (2018), in order to economize on state variables, we approximate the notional account balance at retirement using z_{i64}.[14]

The second pillar of the pension system is a defined-contribution pension account. This account is also income based but the investor can choose how to allocate between bonds and stocks. We denote the return on this asset, which depends on the allocation and the aggregate equity return by R^B_{t+1}. It features mandatory contribution rates $\lambda_{it} \geqslant 0$, and these are the contribution rates we wish to design. Before retirement ($t \leqslant 64$), the law of motion for the DC account balance B_{it} is

$$B_{it+1} = (B_{it} + \lambda_{it} Y_{it}) R^B_{t+1}. \tag{22}$$

Upon retirement at age 65, the DC account and the notional account are converted into two actuarially fair life-long annuities. They insure against longevity risk through within-cohort transfers to survivors from individuals who die. The notional account provides a fixed annuity with a guaranteed minimum, $\underline{Y}$. If the balance of the account is lower than required to meet the guaranteed level at age 65, we let the individual receive the remainder at age 65 in the form of a one-time transfer from the government, which is annuitized as well. The annuity from the DC account is variable and depends on the choice of the equity exposure as well as realized returns. In expectation, the individual will receive a constant payment each year. The conversion from account balances to actuarially fair lifelong annuity payments are functions denoted by $h^B(.)$ and $h^N(.)$. They take the respective balances as arguments. The law of motion for B_{it} after retirement ($t > 64$) is given by

$$B_{it+1} = (B_{it} - h^B(B_{it})) R^B_{t+1} \tag{23}$$

[14]This approximation works well – in simulations the correlation between the actual balance and the approximated one is 0.89.

and similarly for N_{it}.

The third pillar of retirement savings is financial wealth, which is an account outside the pension system that is accessible at any time. Each individual chooses freely how much to save and withdraw from it. In contrast, the contributions to the two pension accounts during the working phase are mandated by the pension policy (rather than by the individual) and are accessible only in the form of annuities during the retirement phase. Importantly, in contrast to the two pension accounts, financial wealth does not include insurance against longevity risk.

The individual starts the first year of the working phase with financial wealth, A_{i25}, outside the pension system. The log of initial financial wealth is distributed $\mathrm{N}(\mu_A - \sigma_A^2/2, \sigma_A^2)$. In each subsequent year, the individual can freely access her financial wealth, make deposits, and choose the fraction to be invested in risk-free bonds and in the stock market. However, the individual cannot borrow:

$$A_{it} \geqslant 0, \tag{24}$$

and the equity share is restricted to be between zero and one,

$$\alpha_{it} \in [0, 1]. \tag{25}$$

Taken together, (24) and (25) imply that individuals cannot borrow at the risk-free rate and that they cannot short the stock market nor take leveraged positions in it.

To enter the stock market outside the pension system, the individual must pay a one-time participation cost, κ_i. (The financial wealth and the decision to invest in the stock market are described later.) A one-time entry cost is common in portfolio-choice models (see, e.g., Alan, 2006; Gomes and Michaelides, 2005, 2008).

The state variable, I_{it}, tracks whether stock market entry has occurred between age 25 and age t; its initial value is zero (i.e., $I_{i25} = 0$). The law of motion for I_{it} is given by

$$I_{it} = \begin{cases} 1 \text{ if } I_{it-1} = 1 \text{ or } \alpha_{it} > 0 \\ 0 \text{ otherwise} \end{cases}, \tag{26}$$

where α_{it} is the fraction of financial wealth invested in the stock market. The cost associated with stock market entry then becomes $\kappa_i(I_{it} - I_{it-1})$.

We allow for different costs for different investors. We assume a uniform distribution of the cost:

$$\kappa_i \sim U(\underline{\kappa}, \bar{\kappa}), \tag{27}$$

where $\underline{\kappa}$ and $\bar{\kappa}$ denote the lowest and highest costs among all investors, respectively. We justify the dispersion in cost with reference to the documented heterogeneity in financial literacy and financial sophistication (see Lusardi and Mitchell, 2014, for an overview). By introducing a cost distribution, we can replicate the fairly flat life-cycle participation profile in the data.[15] On average, low-cost investors will enter early in life, whereas high-cost investors will enter later or never at all. With a sufficiently low value of $\underline{\kappa}$, some low-cost investors will enter immediately. At the end of life, more high-cost than low-cost investors will remain non-participants. For simplicity, we assume that the cost is independent of other characteristics.

Budget constraint

The budget constraint at all stages in life is:

$$C_{it} + A_{it} + \kappa_i(I_{it} - I_{it-1}) = X_{it}, \tag{28}$$

where X_{it} denotes (liquid) cash-in-hand. The law of motion for X_{it} is

$$X_{it+1} = A_{it}R^a_{t+1} + \hat{Y}_{it+1} \tag{29}$$

$$\hat{Y}_{it+1} = \begin{cases} Y_{it+1}\text{exp}(\omega_{it+1}) - \lambda_{it+1}Y_{it+1} & \text{if } t < 64 \\ h^B(B_{it+1}) + h^N(N_{it+1}) & \text{otherwise} \end{cases}, \tag{30}$$

where ω_{it+1} is an idiosyncratic expense shock distributed $\text{N}(-\sigma^2_\omega/2, \sigma^2_\omega)$. Cash-in-hand is comprised of two components. During working life, cash-in-hand consists of liquid financial assets and labor income after expense shocks and mandatory contributions to the DC account.[16] During retirement, it contains liquid financial assets and annuity payments from the two pension accounts instead.

4.2 The individual's problem

Next, we describe the individual's problem. To simplify the notation, we suppress the subscript i. Let $V_t(X_t, B_t, z_t, \kappa, I_t)$ be the value of an individual of age t with cash in hand X_t, DC account balance B_t, a persistent income component z_t, cost κ, and stock market participation experience I_t.

[15] Fagereng et al. (2015) present an alternative setup to account for the empirical life-cycle profiles on portfolio choice. Their setup involves a per-period cost and a loss probability on equity investments.

[16] Notice that, in Equation (30), we do not subtract the notional contribution (λ^N) from income $Y_{i,t+1}$ since $Y_{i,t+1}$ is defined as net of the notional account contribution.

The participant's problem

An individual who has already entered the stock market solves the following problem:

$$V_t\left(X_t, B_t, z_t, \kappa, 1\right) = \max_{A_t, \alpha_t} \left\{ \left((X_t - A_t)^{1-\rho} + \beta \phi_t \mathcal{R}_t \left(V_{t+1}\left(X_{t+1}, B_{t+1}, z_{t+1}, \kappa, 1\right)\right)^{1-\rho} \right)^{\frac{1}{1-\rho}} \right\}$$

subject to equations (18)–(23).

The entrant's problem

Let $V_t^+ \left(X_t, B_t, z_t, \kappa, 0\right)$ be the value for an individual with no previous stock market participation experience who decides to participate at t. This value can be formulated as

$$V_t^+\left(X_t, B_t, z_t, \kappa, 0\right) = \max_{A_t, \alpha_t} \left\{ \left((X_t - A_t - \kappa)^{1-\rho} + \beta \phi_t \mathcal{R}_t \left(V_{t+1}\left(X_{t+1}, B_{t+1}, z_{t+1}, \kappa, 1\right)\right)^{1-\rho} \right)^{\frac{1}{1-\rho}} \right\}$$

subject to equations (18)–(23).

The non-participant's problem

Let $V_t^- \left(X_t, B_t, z_t, \kappa, 0\right)$ be the value for an individual with no previous stock market participation experience who decides not to participate at t. This value can be formulated as

$$V_t^-\left(X_t, B_t, z_t, \kappa, 0\right) = \max_{A_t} \left\{ \left((X_t - A_t)^{1-\rho} + \beta \phi_t \mathcal{R}_t \left(V_{t+1}\left(X_{t+1}, B_{t+1}, z_{t+1}, \kappa, 0\right)\right)^{1-\rho} \right)^{\frac{1}{1-\rho}} \right\}$$

subject to equations (18)–(23).

Note that as $\alpha_t = 0$, the return on financial wealth is simply R_f.

Optimal stock market entry

Given the entrant's and non-participant's problems, the optimal stock market entry is given by

$$V_t\left(X_t, B_t, z_t, \kappa, 0\right) = \max\left\{V_t^+\left(X_t, B_t, z_t, \kappa, 0\right), V_t^-\left(X_t, B_t, z_t, \kappa, 0\right)\right\}.$$

A novel feature of our model is the design of the DC account's contribution rates. We discuss this component in detail following the calibration, which is based on the current constant contribution rates policy.

Table 5: Calibration—model parameters

	Notation	Value
Preferences and stock market entry cost		
Discount factor*	β	0.941
Elasticity of intertemporal substitution	$1/\rho$	0.500
Relative risk aversion*	γ	14
Ceiling for stock market entry cost*	$\overline{\kappa}$	29,250
Floor for stock market entry cost*	$\underline{\kappa}$	0
Returns		
Gross risk-free rate	R_f	1.00
Equity premium	μ	0.04
Standard deviation of stock market return	σ_ε	0.18
Labor income, expense shock, and financial wealth		
Standard deviation of idiosyncratic labor income shock	σ_η	0.072
Weight of stock market shock in labor income	θ	0.040
Standard deviation of idiosyncratic expense shock	σ_ω	0.101
Standard deviation of initial labor income	σ_z	0.366
Standard deviation of initial financial wealth	σ_A	1.392
Mean of initial financial wealth	$\exp(\mu_A - \sigma_A^2/2)$	112,500
Floor for notional pension	$\underline{Y}$	85,829
Contribution rates in pension accounts		
DC account	λ	6.54%
Notional account	λ^{N}	14.95%
Life-cycle profiles		
Labor-income profile (scaled by 1.07)	g_t	**
Survival rates	ϕ_t	***

Note: The table presents the parameter values of the model. * The parameter value has been determined endogenously by simulation of the model. ** The labor-income profiles are discussed in detail in the main text. *** The survival rates are computed from unisex statistics provided by Statistics Sweden.

4.3 Calibration

In this section, we describe our calibration strategy. Table 5 reports the values of key parameters. Most parameters are set either according to the existing literature or to match Swedish institutional details; these parameters can be said to be set exogenously. Four parameters are set to match the data as well as possible; these parameters can be said to be determined endogenously.

Exogenous parameters

There are six sets of exogenous parameters.

First, we set the elasticity of intertemporal substitution to 0.5, which is a common value in life-cycle models of portfolio choice (see, e.g., Gomes and Michaelides, 2005).

Second, we set the equity premium to 4% and the standard deviation of the stock market return to 18%. These choices are in the range of commonly used parameter values in the literature. We set the risk-free rate to zero, which in other calibrations is often set to 1–2%. We argue that this is correct in our model as labor income does not include economic growth. By doing so, we deflate the account returns by (expected) economic growth to obtain coherent replacement rates. As replacement rates in our model are a function of returns rather than a function of final labor income, this choice is more important in our model than in previous ones. Simulations of the labor income process and contributions to the pension accounts validate our strategy. These simulations indicate that replacement rates at age 65 relative to labor income at age 64 are coherent with the Swedish Pension Agency's forecasts.

Third, we set labor income and the parameters of the pension system according to Swedish data. Appendix C contains a detailed description of the Swedish pension system. The level of the income profile (g_t) is first set to match observed labor income. Then the profile is adjusted further to account for the fact that observed labor income in the data is after deductions of DC plan contributions. Typical contribution rates are 7%—the sum of the premium pension account with a contribution rate of 2.5% and the occupational pension account with a typical contribution rate of 4.5%.[17] We therefore scale up the income profile by a factor of 1.07. Following Carroll and Samwick (1997), we estimate the riskiness of labor income. To abstract from other transfers of the welfare state, progressive taxation, etc., we estimate the risk on disposable income. We find that the standard deviation of permanent labor income (σ_η) equals 0.072 and that the standard deviation of transitory risk is 0.101. We use this value for our expense shock (σ_ω). We set the one-year correlation between permanent income growth and stock market returns to 10%. This corresponds to a θ of 0.040. We approximate the distribution of initial labor income and financial wealth using log-normal distributions. The mean financial wealth for 25-year-old default investors is set to SEK 112,500. The cross-sectional standard deviations are set to 0.366 (σ_z) and 1.392 (σ_A) to match the data for 25-year-old individuals.

Fourth, we match the contribution rates to Sweden. As mentioned before, the total contribution rate to DC accounts is 7% of observed labor income. This corresponds to a contribution rate in the

[17] This corresponds to the ITP1 pension plan for birth cohorts 1979 and younger but abstracting from the increase in contributions above the cap of the notional account, which is intended to offset a cap on contribution rates to the DC and notional accounts.

Table 6: Matched Moments in Data and Model

	Data	Model
Financial wealth-to-labor income ratio	0.92	0.94
Stock market participation	0.51	0.49
Equity share (conditional)	0.44	0.42

Note: The table presents matched moments in the data and model.

model of 6.54% (0.07/1.07). The contribution rate in Sweden for the notional account is 16% of observed labor income. This corresponds to 14.95% in the model (0.16/1.07).

Fifth, we determine the annuity divisor for the notional account in retirement. We use the unisex mortality table of Statistics Sweden to determine ϕ_t. We assume that the notional account continues to be invested in the risk-free bond and allow for inheritances within a cohort from dying to surviving individuals, incorporating these into the returns of the survivors. We then use the standard annuity formula to reach an annuity factor of 5.6% out of the account balance at age 65. We use the same formulas for the DC account, though we adjust the expected return to the endogenous choice of the DC equity share in retirement.

Finally, we calibrate the DC equity share profile. We use the glide path 100-minus-age, which is a very common allocation and similar to the default fund in the premium pension plan.

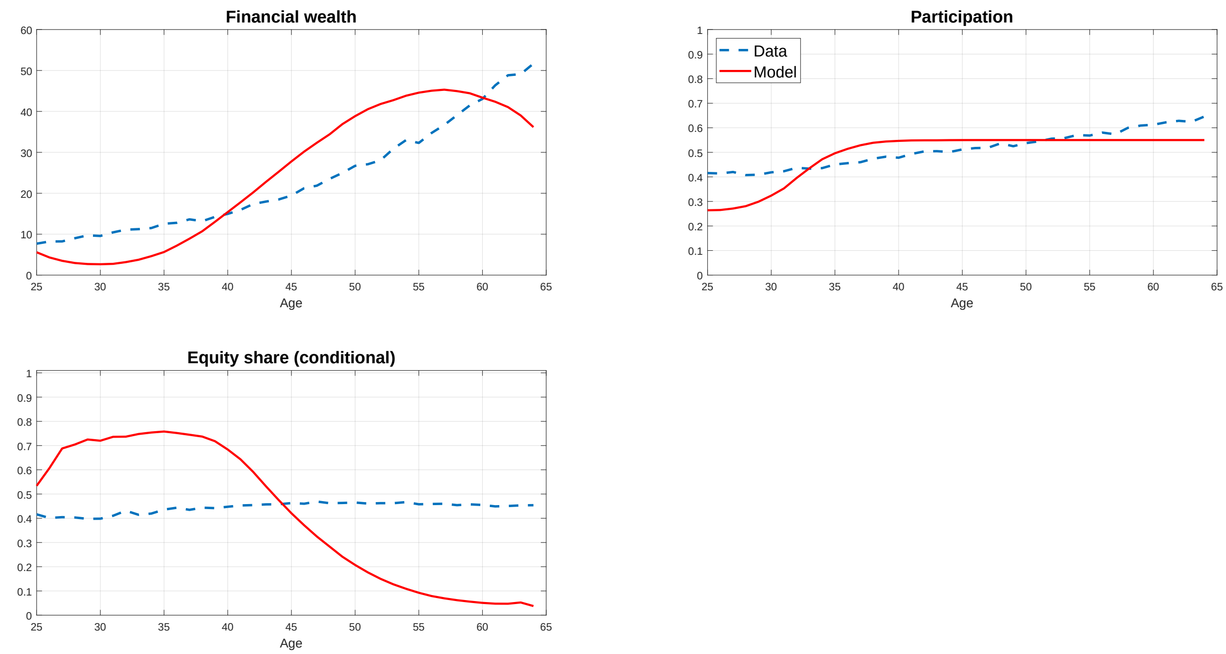
Endogenous parameters and model fit

Four parameters are treated as endogenous in the calibration. We consider data from the working phase.[18] Table 6 shows the close fit between data and model moments. The discount factor (β) is calibrated as 0.941 to match the average ratio of financial wealth to labor income during the working phase (0.94 in the model and 0.92 in data). The top-left panel of Figure 3 shows the full life-cycle profile of financial wealth. The model fits the financial wealth quite well up to age 60 and undershoots after that.

The support of the cross-sectional distribution of participation costs is set so that we match the average stock market participation rate between ages 25 and 64. As can be seen in the top-right panel of Figure 3, participation is almost flat over the life cycle. Intuitively, the parameter

[18]Note that we match the model to data from 2007. This does not allow us to extract cohort or time effects as in, e.g., Ameriks and Zeldes (2004). However, Vestman (2019) finds that cohort and time effects are not strongly present in the data.

Figure 3: Calibration



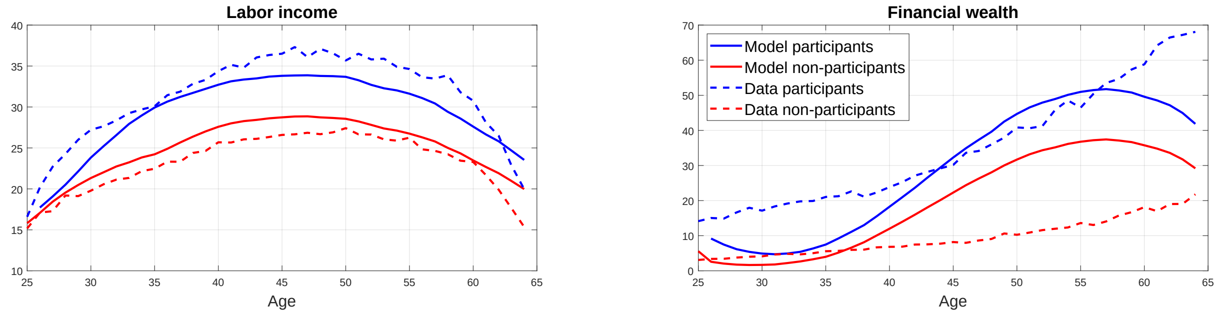
Note: The figure shows the life-cycle properties of the variables that the calibration targets (targets are their average levels). Financial wealth is expressed in SEK 10,000s.

$\underline{\kappa}$ affects the participation rate among the young, who are poor in terms of financial wealth and reluctant to enter the stock market if the cost is high. The relatively high participation rate of young individuals therefore leads us to set $\underline{\kappa} = 0$. The parameter $\bar{\kappa}$ is then determined to match the average participation rate from age 25 to 64, which is 0.49 in the model and 0.51 in the data. We obtain this participation rate by setting $\bar{\kappa} = 29,250$. As the distribution is uniform, this corresponds to an average participation cost of SEK 14,625. We find our modeling approach appealing as the uniform distribution of the cost enables the model to replicate the flat participation profile in the data.[19]

Finally, the conditional equity share determines the relative risk aversion coefficient γ. We weigh each age group's equity share equally. A relative risk aversion of 14 provides a good fit. The conditional equity share is 0.42 in the model and 0.44 in the data. The lower-left panel of Figure 3 depicts the life-cycle profile. As is common in life-cycle models such as ours, the model overshoots the data when financial wealth is low and undershoots when liquid financial wealth is high. We are reluctant to increase the relative risk aversion above 14, as this would lead to a worse discrepancy close to retirement age.

[19] Technically, we approximate the uniform distribution using five equally weighted discrete types (the five costs are equally spaced between zero and SEK 29,250).

Figure 4: Model fit



Note: The figure shows labor income and financial wealth conditional on stock market participation. Financial wealth is expressed in SEK 10,000s.

Figure 4 shows that the distribution of entry costs produces an endogenous sorting of individuals into stock market participants and non-participants that matches the data well. The left panel shows that the average labor income by participation status is similar in the model and the data. The right panel shows the financial wealth in the model and in the data. The sorting by financial wealth to participants and non-participants is consistent with the data but weaker.[20] Financial wealth in the model peaks just before retirement, somewhat earlier than in the data.

Replacement rates in the benchmark pension system

We compute the replacement rate out of the DC account in the benchmark and its cross-sectional dispersion, reported in column (1) of Table 7. The mean across individuals is 0.29 with substantial cross-sectional dispersion. The standard deviation is 0.12 and percentiles 95 and 5 are 0.54 and 0.15, respectively. The Swedish Pensions Agency has reported similar dispersion in replacement rates (Pensionsmyndigheten (2021)). This cross-sectional dispersion translates into considerable dispersion in wealth at 65. Panel B of Table 7 considers the thought experiment that all three accounts would be annuitized at 65 (i.e., including financial wealth). It would yield a mean replacement rate of 0.94 with a standard deviation of 0.30 and a 95th percentile corresponding to 1.57.

These replacement rates can be contrasted to the wealth dispersion if there had been no DC pension plan, which is reported in column (2). In this setting, wealth accumulation is lower, resulting in a mean replacement rate of 0.80. It is, however, noteworthy that the cross-sectional dispersion in replacement rates out of wealth is considerably smaller, with a smaller standard deviation of 0.20

[20] It is well known that it is difficult to generate wealth inequality in life-cycle models with incomplete markets. This has been addressed by incorporating heterogeneity in discount factors (Krusell and Smith, 1998) or a right-skewed income process (Castaneda et al., 2003).

Table 7: Replacement rates and welfare gains

	(1)	(2)	(3)	(4)	(5)
	Benchmark	No DC	Age dependency	B/Y dependency	Both age and B/Y dependency
Panel A: Replacement rate out of the DC account					
Mean	0.29	—	0.29	0.29	0.29
Standard deviation	0.12	—	0.10	0.08	0.07
Percentile 95	0.54	—	0.49	0.45	0.43
Percentile 5	0.15	—	0.17	0.19	0.20
Panel B: Replacement rate out of total wealth					
Mean	0.94	0.80	0.93	0.93	0.92
Standard deviation	0.30	0.20	0.29	0.28	0.27
Percentile 95	1.57	1.19	1.54	1.52	1.50
Percentile 5	0.65	0.60	0.65	0.65	0.65
Panel C: Welfare gain relative to benchmark (in percent)					
Mean	—	5.1	1.1	1.2	1.8
Standard deviation	—	0.7	0.2	0.2	0.3
Percentile 95	—	6.0	1.4	1.4	2.0
Percentile 5	—	3.9	0.7	0.7	1.3

Note: Panel A reports moments of replacement rates out of the DC account ($h^B(B_{65}/Y_{64})$. Panel B reports moments of replacement rates out of total wealth ($(h^B(B_{65} + A_{65}) + h^N(N_{65}))/Y_{64}$). Panel C reports moments of ex ante welfare gains associated with a shift from the benchmark to each one of the other DC plan designs. The age-dependent policy in column (3) corresponds to $\lambda_{it} = 0.0104 + 0.003t$. The B/Y-dependent policy in column (4) corresponds to $\lambda_{it} = 0.0645 - 0.2\left(\frac{B_{it}}{Y_{it}}/\chi_t - 1\right)$. The policy in column (5) corresponds to $\lambda_{it} = 0.0101 + 0.003t - 0.15\left(\frac{B_{it}}{Y_{it}}/\chi_t - 1\right)$.

and a 95th percentile of 1.19. Imposing mandatory pension savings, in the benchmark environment in the form of a constant fraction of income, therefore increases the dispersion of replacement rates.

5 Flexible DC contribution rates

We now design a DC pension plan with a flexible contribution rate that follows the principles of consumption-savings theory. We have shown that optimal contribution rates should increase with age and decrease in the balance-to-income ratio. We therefore consider flexible DC contribution rates of the form:

$$\lambda_{it} = \beta_0 + \beta_1 t + \beta_2 \left(\frac{\frac{B_{it}}{Y_{it}}}{\chi_t} - 1 \right), \tag{31}$$

where t indicates the individual's age (minus 24) and $\frac{B_{it}}{Y_{it}}$ indicates the individual's balance-to-income ratio, which is compared to the target balance-to-income ratio χ_t for individuals of age t.[21] Note that we do not consider early withdrawals from the DC account and hence apply a lower bound to contribution rates of 0.

The choice of functional form deserves discussion. First, it nests the current benchmark pension system: if $\beta_1 = \beta_2 = 0$, the contribution rate λ_{it} simplifies to a constant contribution rate β_0 for all investors, irrespective of their age or their balance-to-income ratio. Letting β_1 and/or β_2 be non-zero allows the contribution rate to adjust according to the two principles of optimal consumption-savings behavior. Second, we chose a linear age profile to capture the effect of increasing contribution rates with age. In principle, we could allow for higher order polynomials to improve the fit to the income profile of the population of workers. However, since we already obtain large welfare improvements with the linear rule—and in order to keep the design as simple as possible—we chose to restrict our analysis to this specification. Finally, the adjustment due to the balance-to-income ratio is a linear function of the centered percentage deviations from the target ratio. This functional form has three advantages. First, it gives the parameter β_2 a clear interpretation: It is the semi-elasticity of the contribution rate to the balance-to-income ratio if an investor currently is on target. Second, for $\beta_2 < 0$ (which will be the case of interest), $|\beta_2|$ is the upper bound for how much the contribution rates can theoretically be increased due to the balance-to-income ratio (for investors with $\frac{B_{it}}{Y_{it}} = 0$, i.e., no savings in their DC account). This precludes excessive contribution rates even for extreme shock realizations. Third, the functional form assumption also ensures that the elasticity of disposable income ($Y_{it}^{disp} = (1 - \lambda_{it})Y_{it}$) to gross income is strictly positive under the following condition (see Appendix D for the proof):

$$\frac{\partial Y_{it}^{disp}}{\partial Y_{it}} \cdot \frac{Y_{it}}{Y_{it}^{disp}} \geqslant 0 \ \ \forall i, t \qquad \text{iff} \qquad \beta_0 + \beta_1 t - \beta_2 \leqslant 1. \tag{32}$$

This condition is easily satisfied for all cases of interest. Thus, our choice of functional form ensures that the contribution rates are well-behaved for the whole state space.

We perform a grid search over the parameters $(\beta_0, \beta_1, \beta_2)$ and compute the maximum welfare

[21] The adjustment for an age-specific target replacement rate is necessary in a setup with more than two working-life periods since the average account balance naturally increases with age. In contrast, in the theoretical model of section 2, there was only one period where contribution rates were a function of the balance-to-income ratio. In the theory section, we could thus abstract from this adjustment.

gain relative to the benchmark calibration.[22] We impose several restrictions in the search. A common restriction in all our searches is that we require the average replacement rate out of the DC account to be maintained at a certain level, for instance 0.29 if we wish to target the DC replacement rate in the benchmark setting. Moreover, we only consider parameter combinations that ensure that the average contribution rate does not exceed 15% at any age. The reason is that these DC-account contributions are made in addition to the contributions to the notional account of $\lambda^N = 14.95\%$.

To facilitate comparisons and illustrations of mechanisms, we consider four subsets of contribution rates[23]:

1. Constant contribution rates. We impose $\beta_1 = \beta_2 = 0$ and vary β_0 between the contribution rate of the benchmark and the case of the No-DC plan.

2. Age-dependent contribution rates. We impose $\beta_2 = 0$ and then adjust β_0 so that for each value of β_1 we achieve a specific average replacement rate out of the DC account.

3. Balance-to-income (B/Y) dependent contribution rates. We impose $\beta_1 = 0$ and determine the target balance-to-income ratios $\{\chi_t\}_{t=25}^{64}$ so that the average contribution rate is constant over the life cycle.

4. Age and balance-to-income dependent contribution rates. In this case, we impose that the average contribution rate for each age group is equal to $\beta_0 + \beta_1 t$ to facilitate the interpretation of the role of the adjustment due to the balance-to-income ratio.

5.1 Welfare effects and dispersion of replacement rates

Columns (3) to (5) of Table 7 report our findings from grid searches over sets 2, 3, and 4, imposing that the average replacement rate out of the DC account should be equal to the benchmark. Panels A and B report the resulting moments for replacement rates out of total wealth and out of the DC account alone, respectively. Panel C shows the associated welfare gains. The welfare gain statistics of a shift to the No-DC setting are reported in column (2). The average ex ante welfare gain of the No-DC setting is substantial at 5.1 percent. Notably, in expectation nobody loses from abolishing the DC plan. This is because the insurance value against longevity that the DC plan offers is insufficient to outweigh the rigidity during working life. We will use the gains of moving to a No-DC setting as a yardstick when we evaluate more flexible designs of the contribution rate.

[22] With Epstein-Zin utility, it is straightforward to compute the consumption equivalent. It is proportional to the value function. Our reference to ex ante welfare means that we use the value functions of the 25-year-olds.

[23] See Appendix E for a detailed description of our search algorithm.

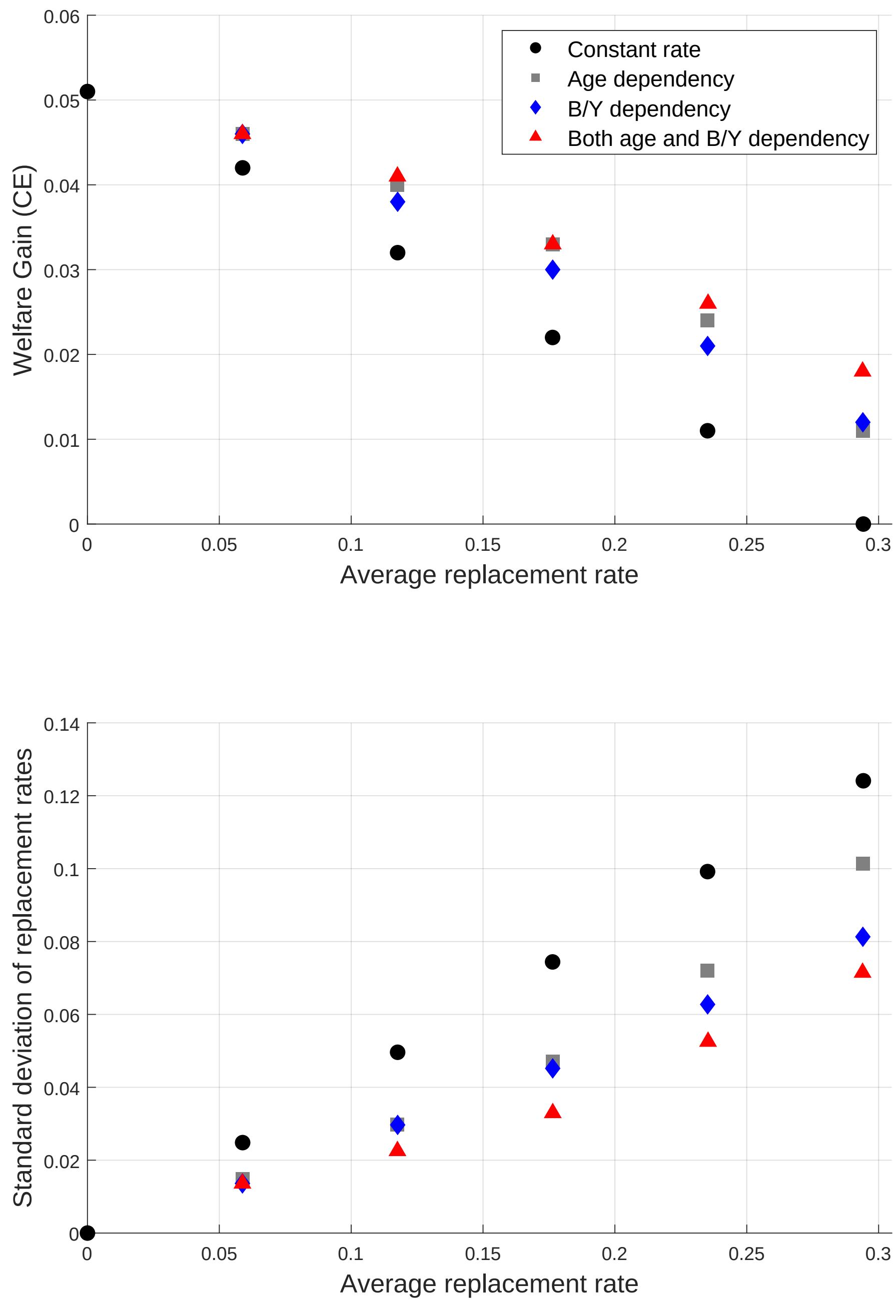
The age-dependent contribution rate that maximizes welfare is $\lambda_{it} = 0.0104 + 0.003\, t$. This formula implies that contribution rates start low (about 1.3% at age 25) and then increase by 0.3 percentage points per year. At 64, the contribution rate peaks at 13% percent. As reported in Panels A and B, this age-dependent contribution rate is able to reduce the cross-sectional dispersion in replacement rates to 0.10 and limits the 95th percentile to 0.49. This results in an average welfare gain of 1.1 percent relative to the benchmark. Thus, it bridges 22 percent of the welfare gap between the benchmark and the No-DC plan. Intuitively, this is achieved by improving individuals' ability to self-insure early in life when their marginal utility is high as well as reducing the undesired dispersion of the replacement rate out of the DC account.

The balance-to-income dependent contribution rate that maximizes welfare is $\lambda_{it} = 0.0645 - 0.2\left(\frac{B_{it}}{Y_{it}}/\chi_t - 1\right)$. This formula implies that an investor who falls short by 1 percent from the balance-to-income target should increase her contribution rate by 0.2 percentage points. A fall in income or an increase in the account balance hence leads to cash-flow benefits. Column (4) of Table 7 reports the associated statistics. According to Panel B, this policy is able to reduce the cross-sectional dispersion in replacement rates even more than the age-dependent policy. The standard deviation is a mere 0.08. Panel C reports welfare gains. Interestingly, at 1.2 percent, this rule for the contribution rate is associated with a slightly higher welfare gain than the best age-dependent contribution rate.

After examining each instrument separately, we now describe our proposal, which combines both instruments. The best combination is $\lambda_{it} = 0.0101 + 0.003\, t - 0.15\left(\frac{B_{it}}{Y_{it}}/\chi_t - 1\right)$. According to that formula, average contribution rates increase by 0.3 percentage points for every year of age and the adjustment based on the balance-to-income ratio implies a semi-elasticity of -0.15, which is slightly weaker than in the pure balance-to-income rule. The results are reported in column (5). Panel B reports a further decline in the cross-sectional distribution, with a standard deviation of only 0.071. Panel C reports the welfare gains. Interestingly, this rule adds an additional welfare benefit, which implies that the two instruments complement each other. The average welfare gain is 1.8 percent. Thus, the best rule for the contribution rate covers 36 percent of the gap between the benchmark setting and the No-DC setting.

To gauge the role of flexible contribution rate rules versus target replacement rates, Figure 5 reports the outcome of a broader grid search, targeting different DC replacement rates. The frontier of dots represents the designs with successively smaller constant contribution rates. Those correspond to 0.2, 0.4, 0.6 and 0.8 of the actual replacement rate target that we aimed at so far. The top panel depicts welfare gains relative to the benchmark economy. As before, the No-DC case serves as a yardstick, corresponding to a replacement rate of zero and a welfare gain of 5.1 percent. The gray squares depict the possible welfare gain from age-only dependencies for the same average

Figure 5: Replacement rate targets, welfare gains, and dispersion of replacement rates



Note: The figure shows welfare gains (consumption equivalents) and the dispersion of replacement rates out of the DC account against target replacement rates for four types of contribution rate policies: (i) constant contribution rates ($\lambda_{it} = \beta_0$), (ii) only age coefficients ($\lambda_{it} = \beta_0 + \beta_1 t$), (iii) only B/Y coefficients ($\lambda_{it} = \beta_0 + \beta_2(\frac{B_{it}}{Y_{it}}/\chi_t - 1)$, and (iv) age and B/Y coefficients ($\lambda_{it} = \beta_0 + \beta_1 t + \beta_2(\frac{B_{it}}{Y_{it}}/\chi_t - 1)$).

replacement rates, and blue diamonds the corresponding the B/Y-only dependencies. The red triangles show the welfare gains achievable with both dependencies for the different replacement rates. One important insight from this figure is how disadvantageous constant contribution rates are relative to our proposals. For all average replacement-rate targets, the three sets of rules (age-only, B/Y-only, and both dependencies) attain a considerable fraction of the gain associated with the No-DC plan. For low average replacement-rate targets, the performance of all three rules is similar. For higher replacement-rate targets, however, the rules that only allow for one of the adjustments are no longer able to achieve the same welfare gains. In contrast, the combined design with both adjustments achieves large welfare benefits even at the high level of replacement rates that are currently implemented in the benchmark design.

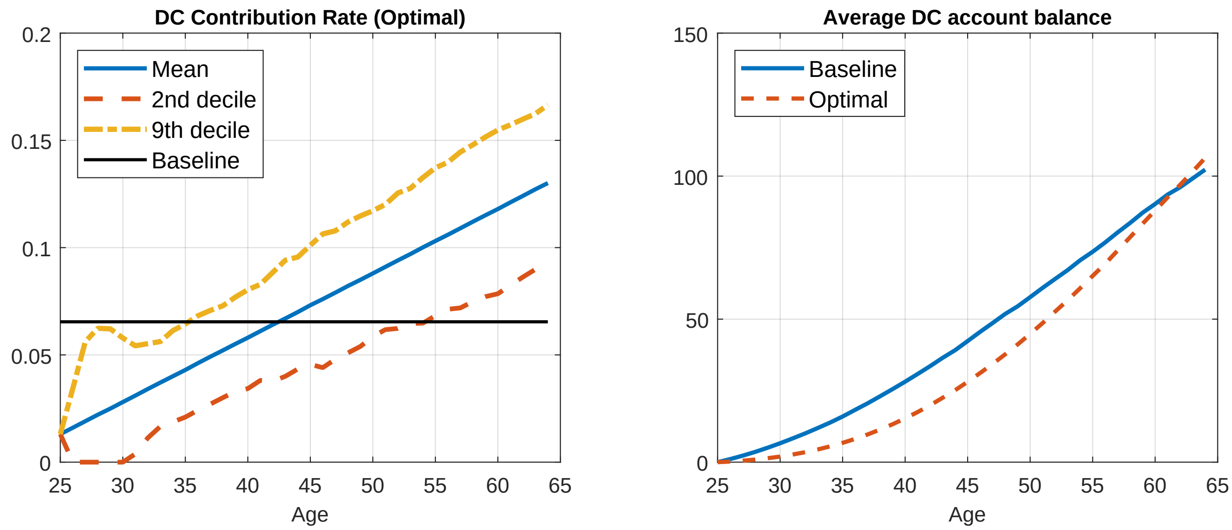
The bottom panel of Figure 5 reports the standard deviation of replacement rates that is associated with the different rules. As the required average replacement rate increases, the associated standard deviation of replacement rates also increases for all types of designs. However, for each target replacement rate, the dispersion can be substantially reduced by redesigning the contribution rates to follow consumption-savings principles. As in the case of the welfare gains, the differential effects across the three rules become particularly pronounced for higher replacement-rate targets: Only the combined design with both dependencies is able to reduce the dispersion of replacement rates consistently by more than 40 percent, even for high replacement-rate targets.

The results from the quantitative model thus confirm that, in a realistically calibrated life-cycle model, the insight from the simple model of section 2 hold: By allowing mandatory contribution rates to follow the principles of optimal consumption-savings theory, it is possible to achieve large welfare gains without changing the average replacement rate that the pension system provides. Moreover, this change substantially reduces the dispersion of replacement rates.

5.2 Impact on investor behavior and mechanisms behind welfare gains

Where do the welfare benefits of the flexible design come from? To shed light on this question, we analyze the effect of our proposed design (the combined rule) on the behavior of investors. Figure 6 illustrates the implications of our proposal on the evolution of contribution rates and DC account balances. The left panel details that contribution rates are low early in life. The average contribution is lower than the one in the benchmark until age 42 and reaches its highest value at thirteen percent just before retirement. At the same time, heterogeneity in contribution rates is the largest early in working life. This is driven by return shocks: The equity share in the DC account follows a glide path of “100 - age”, so for young investors, the DC equity share is the highest. Sequences of extreme return realizations early in working life hence lead to relatively larger swings in the DC account balance and affect the contribution rates relatively more before the balance-to-income adjustment in the contribution rates over time reduces the dispersion again. Despite this larger heterogeneity, even the 9th decile of contribution rates is lower for young investors than the

Figure 6: Benchmark vs. flexible pension system



Note: The left panel shows the average and the 2nd and 9th deciles of the contribution rate into the DC account (i.e., λ_{it}) for the optimal age- and balance-to-income-dependent design compared to the flat contribution rates of the benchmark design. The right panel shows the average DC account balance during working life. Values are expressed in SEK 10,000s.

benchmark flat contribution rate of 6.54 percent (depicted as the horizontal black line). The right panel shows that this increase in contribution rates over life implies that the DC account balance displays more exponential growth under the proposed rule and on average account balances do not reach the level of the benchmark until a few years before retirement.[24]

The impact of our proposed design on the behavior of investors is detailed in Table 8. It shows for different points in working life the average consumption, financial wealth, and stock market participation in the benchmark economy under the proposed flexible design and changes of the flexible design relative to the benchmark (in percent). Due to the lower contribution rates when young, investors are able to consume more in the first half of their working life (see panel A). At age 30, the average consumption is higher by 3.8 percent due to the flexible design. Since overall consumption levels are lower at this point in the life cycle, marginal utility is higher than later in life, where consumption levels are typically higher. This shift of consumption towards younger investors with higher marginal utility is one of the main driving forces of the welfare gains of our proposed design.

Financial wealth (panel B) is somewhat higher early in life (by on average SEK 2,000, or 7.4 percent) and lower at age 60 (by on average SEK 62,000, or 14.3 percent). This is the result of

[24]Figure A.1 in Appendix F shows the plots of the whole distribution of the DC account balance as well as distributions of financial wealth and consumption.

Table 8: Effect of the optimal design on behavior

	Age 30	Age 40	Age 50	Age 60
Panel A: Average Consumption				
Benchmark	20.6	26.4	27.7	25.0
Optimal	21.4	26.5	27.1	24.3
Changes (percent)	3.9	0.4	-2.2	-2.8
Panel B: Average Financial Wealth				
Benchmark	2.7	15.4	38.8	43.4
Optimal	2.9	16.7	38.2	37.2
Changes (percent)	7.4	8.4	-1.5	-14.3
Panel C: Average Stock Market Participation				
Benchmark	0.32	0.55	0.55	0.55
Optimal	0.29	0.39	0.39	0.39
Changes (percent)	-9.4	-29.1	-29.1	-29.1

Note: The table reports average consumption, financial wealth, and stock market participation at different points in the life cycle. It compares the levels in the benchmark calibration and under the optimal design of contribution rates (in SEK 10,000s) and computes the change from baseline to optimal in percent.

two opposing forces.[25] On the one hand, investors have more resources available for saving early in life since their contribution rates are lower. This increases savings. On the other hand, they face lower period-to-period risk since their disposable income is more stable than in the benchmark. Table 9 shows that the standard deviation of changes in disposable income is substantially lower under the flexible design than in the system where contribution rates are flat. The reason is that the balance-to-income adjustment in the contribution rate implies that shocks to gross income are partially smoothed by corresponding changes in the contribution rate: All else equal, when income drops the balance-to-income increases, so that contribution rates fall. The correlation of changes in disposable income with changes in gross income is thus only 0.95 while it is 1.00 in the benchmark. This reduced variation in disposable income implies that the investor has less need for precautionary savings, and as a result the optimal level of financial wealth decreases. This reduction in disposable-income risk is another driver of the welfare benefits from the flexible

[25] Tables A.6 to A.8 in Appendix F show the corresponding results to Tables 8 and 9 for the contribution-rate designs that are only age-dependent or only B/Y-dependent, respectively. The results for these designs depict the mechanisms described below.

Table 9: Changes in disposable income—standard deviation and correlation with shocks

	Standard Deviation	Correlation with Changes in Gross Income	Correlation with Stock Market Returns
Benchmark	21.24	1.00	-0.0014
Optimal	19.26	0.95	-0.0074

Note: The table reports statistics of changes in disposable income: average lifetime standard deviation, correlation with changes in gross income, correlation with changes in stock market returns.

design.[26]

In terms of stock market participation, Panel C of Table 8 shows that the average participation rate is lower under the flexible design for all age groups by 3–16 percentage points (or 9–29 percent) depending on age. This again is the outcome of two opposing mechanisms. On the one hand, financial wealth is higher early in life. All else equal, this increases the incentives to enter the stock market. On the other hand, however, Table 9 shows that the correlation of changes in disposable income with stock returns is higher (in absolute terms). In the benchmark design with constant contribution rates, disposable income is correlated with stock returns only through the correlation of gross income with aggregate shocks. In contrast, under the flexible design, stock returns directly affect contribution rates through the balance-to-income adjustment. This increased correlation of disposable income with returns, together with lower financial wealth later in life, implies that investors optimally participate less in the stock market.

6 Generality and importance of redesigning contribution rates

So far we have argued that contribution rates into mandatory DC pension plans should follow the principles of consumption-savings theory. In order to evaluate how general these results are, we now show their robustness to various sources of potential model misspecification. Moreover, we compare the welfare benefits of redesigning contribution rates with the benefits of redesigning other aspects of DC pension plans to highlight the importance of contribution rates.

[26]Since income and equity shocks are only partially correlated, disposable income is also affected by equity shocks, which in turn affect the balance-to-income ratio. Quantitatively, however, as we see from Table 9, the total variation in disposable income is lower with the optimal rule.

6.1 Temptation and self control

In our main analysis, we have assumed that investors have standard rational preferences. However, going back to Feldstein (1985), a long history of literature justifies the existence of mandatory pension or social security systems with the consideration that investors might have time-inconsistent preferences. In this section, we show that the welfare benefits of our proposed contribution rates are robust even if investors suffer from the temptation to spend all available resources.[27] To maintain the same framework as in our main analysis, we generalize Epstein and Zin (1989) preferences to allow for temptation and self control as in Gul and Pesendorfer (2001, 2004). Specifically, we replace the objective function of an investor at age t (equations (16)-(17)) with the following formulation

$$U_t = \left(C_t^{1-\rho} + \tau(C_t^{1-\rho} - X_t^{1-\rho}) + \beta\phi_t E_t \left[U_{t+1}^{1-\gamma} \right]^{\frac{1-\rho}{1-\gamma}} \right)^{\frac{1}{1-\rho}}, \tag{33}$$

$$U_T = C_T + \tau(C_T - X_T), \tag{34}$$

where τ is the degree of temptation. In this setup, the new term in the utility specification captures the costs of self control. Intuitively, investors are always tempted to maximize their current utility, which they would obtain if they did not save anything and instead spent all available cash on hand on current consumption. However, if they resist this temptation their actual felicity from current consumption falls short of this temptation and they suffer costs of self control. These costs increase the more self control is exerted (the smaller C_t compared to X_t) and the stronger the degree of temptation τ. For $\tau = 0$, the preferences reduce to standard Epstein and Zin (1989) preferences.

To investigate the robustness of our preferred designs to the presence of temptation we proceed in two steps. First, for each degree of temptation τ, we recalibrate the other preference parameters (discount factor β and the ceiling for stock market entry costs $\bar{\kappa}$) to match the target wealth-to-income ratio and the average participation rate in the data.[28] Second, we compute the welfare effects of the designs that we identified in the main analysis. Welfare gains are expressed as the percentage increases in permanent income that would make individuals in the benchmark environment (i.e. constant contribution rates) as well off as they would be in an environment with the alternative contribution rate designs, holding initial cash-on-hand constant.[29]

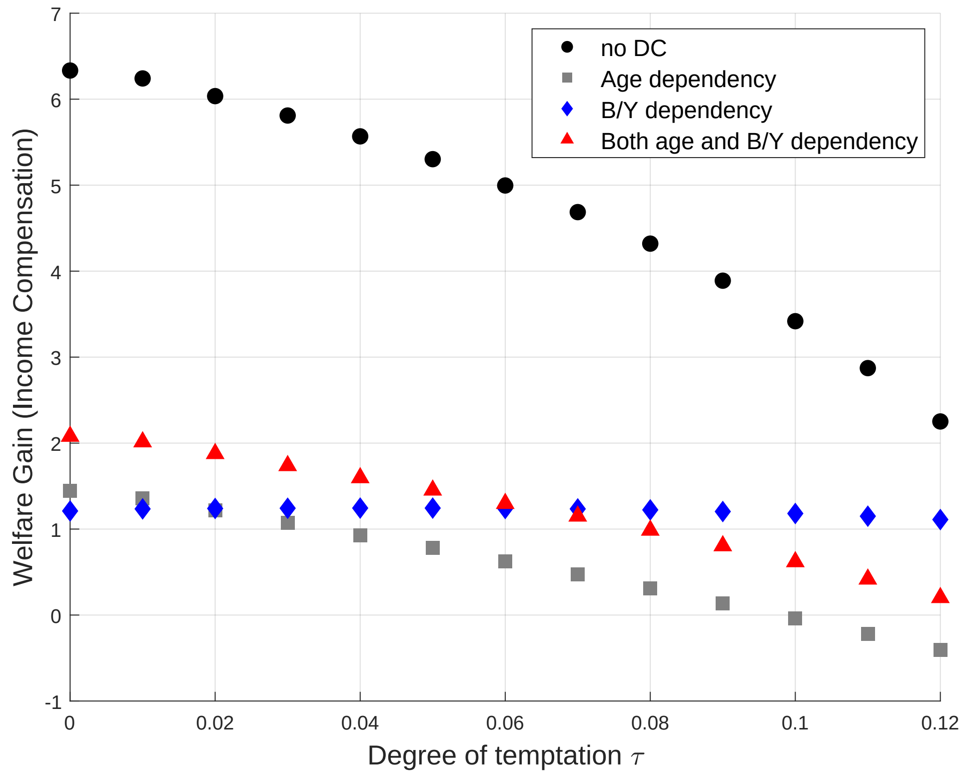
Figure 7 depicts the results. Several things are worth pointing out. First, for $\tau = 0$ both the new

[27] For an alternative preference specification with time inconsistency see the working paper version of the paper (Schlafmann, Setty, and Vestman, 2021).

[28] We assume that risk aversion γ remains at the benchmark value, independent of the degree of temptation.

[29] The introduction of a second welfare criterion is necessary as with recursive preferences and temptation it is no longer possible to obtain closed form solutions for a consumption equivalent.

Figure 7: Temptation and welfare gains



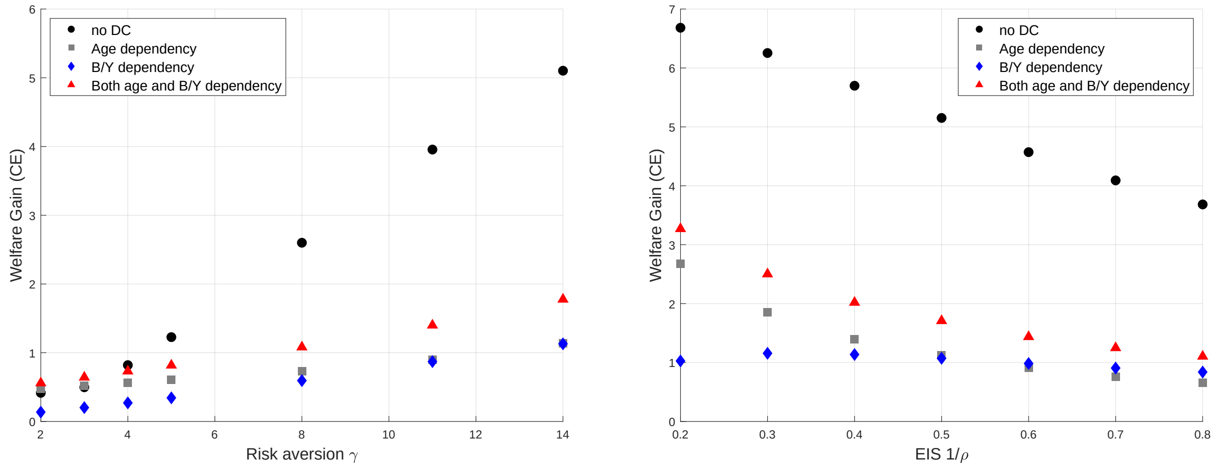
Note: The figure shows the welfare gains (expressed as income compensation) of the alternative designs of contribution rates for different degrees of temptation. Note that, for each degree of temptation, the model has been recalibrated to match data moments (hence, preference parameters β and $\bar{\kappa}$ vary across different horizons).

welfare criterion and the previous welfare criterion are valid. Comparing the income compensation gains shown in figure 7 with the consumption equivalent gains in the main analysis (see table 7) we see that the two criteria are of very similar magnitude (e.g. for the combined rule the income compensation gain is 2.2 percent while the consumption equivalent gain is 1.8 percent).

Second, as is expected if individuals suffer from the temptation to forgo savings, removing the mandate of a DC account becomes less beneficial for higher degrees of temptation. While removing a savings constraint improves flexibility, it at the same time eliminates the commitment to save. It thus increases the temptation to overconsume which increases in importance as the degree of temptation increases. Note that the maximum considered value of the temptation parameter, $\tau = 0.12$, is at the upper end of what is assumed to be plausible values for this parameter (see, e.g., Schlafmann (2021)).

Third, the welfare benefits of age dependency strongly depend on the degree of temptation. As temptation becomes stronger, age dependency naturally becomes less beneficial since lower contribution rates early in life increase the temptation of young workers to overconsume. In contrast,

Figure 8: Preference parameters and welfare gains



Note: The figure shows the welfare gains (expressed as consumption equivalent) of the alternative designs of contribution rates for different degrees levels of risk aversion (left panel) and elasticity of intertemporal substitution (right panel). Note that, for each specified parameter value, the model has been recalibrated to match data moments (hence, preference parameters β and $\bar{\kappa}$ vary across different horizons).

the balance-to-income dependency remains equally beneficial across all degrees of temptation. As discussed earlier, this rule reduces the variance of disposable income, which is welfare-improving irrespective of temptation. Taken together, the combined design achieves welfare gains for all considered degrees of temptation. However, if the degree of temptation is strong enough, then the combined design no longer dominates the balance-to-income-only design. Across all degrees of temptation, redesigning contribution rates (either as combined rule or balance-to-income only) can achieve welfare benefits between one third and one half of the welfare potential of removing the mandated constraint.

We therefore conclude that the recommendation to include the two basic principles into the contribution rate design is robust to allowing for potential problems of self control in investor preferences. In addition, if a policy maker is worried about strong degrees of temptation, then more emphasis should be put on the balance-to-income adjustment than on the age dependency.

6.2 Specification of Preference Parameters

A second source of potential model misspecification is the choice of calibration targets and associated calibrated parameter values. In the main analysis, we target the average equity share of stock market participants and obtain a value for the risk aversion parameter of 14. This is at the upper end of what is typically considered. We therefore investigate how sensitive our results are to different levels of risk aversion. We recalibrate the model for various parameter values of risk aversion, which means that the time discount factor β and the participation costs κ adjust to match the average asset-to-income ratio and the average participation rate during working life. We then compute the welfare gains of our proposed designs for each parameter constellation.

Figure 8 (left panel) displays the results. While the absolute size of the welfare gains becomes smaller with decreasing risk aversion, all proposed designs are welfare improving for all considered parameter values. Notice that the welfare gains of abolishing the DC plan become very small for low levels of risk aversion. The reason is that abolishing the DC plan not only removes the constraint of mandatory savings. At the same time, it also abolishes the annuities from the pension plan during retirement and hence the insurance against longevity. To match the same target asset-to-income ratio, the time discount factor β has to increase as risk aversion decreases. And for more patient individuals, longevity insurance at the end of life becomes more valuable. The total welfare gain of removing DC plan is thus smaller. In contrast, our proposed rules merely redesign the contribution rates but maintain the annuities and hence the longevity insurance during retirement.[30]

Another potentially important preference parameter is the elasticity of intertemporal substitution (EIS). As with risk aversion, we recalibrate the model for various parameter values of EIS and calculate the welfare gains of our proposed rules. Figure 8 (right panel) shows that redesigning contribution rates is welfare improving for all considered value of the EIS. Comparing the level of welfare gains across the range of EIS, we find that the welfare potential of redesigning contribution rates is larger for lower levels of EIS (the parameter value in our main analysis is 0.5). The more inelastic the preferences, the more individuals gain from the freedom to hold their savings in liquid form and hence to be able to use for consumption smoothing across periods. This explains the increasing benefits from removing the DC plan. Moreover, it implies that relieving individuals from saving early in life, and hence allowing them to build up a buffer stock of savings, is particularly valuable. Both the combined rule and the age-only rule therefore lead to increasing welfare benefits the lower the EIS. Finally, the balance-to-income adjustment is equally beneficial across the whole range of possible values for EIS.

Taken together, we thus conclude that redesigning contribution rates in line with the two basic principles of consumption-savings theory robustly increases welfare for a wide range of possible preference parameters. Our results are thus robust to this source of potential model misspecification.

[30] In appendix figure A.2, we repeat the current analysis in a counterfactual scenario where there is no longevity insurance, neither in the benchmark economy nor in any of the proposed designs (the DC balance is paid out lump sum at entry to retirement in all scenarios). This analysis thus isolates the welfare effects of redesigning the contribution rates without affecting insurance during retirement. The results support our interpretation.

Table 10: Welfare gains of changing alternative policy margins

		Mandatory equity share		Annuitization Fraction	
	Both age and B/Y dependency	constant	DSV (2018)	no annuities	50% annuitized
	(1)	(2)	(3)	(4)	(5)
Mean	1.8	-0.02	0.05	-0.01	0.04
Standard deviation	0.3	0.01	0.04	0.04	0.01
Percentile 95	2.0	-0.01	0.07	0.03	0.05
Percentile 5	1.3	-0.04	0.02	-0.08	0.03

Note: The table reports moments of welfare gains (in percent of consumption equivalent) for policies that change alternative margins of mandatory DC pension plans. For comparison, column 1 repeats the welfare effects of our preferred rule for contribution rates (the combined rule). Columns 2-3 show the welfare effects changing the investment: a constant mandatory equity share throughout working life (column 2) and the optimal mandatory equity share according to Dahlquist, Setty, and Vestman (2018) (column 3)). Columns 4-5 vary the fraction that of the DC balance that is annuitized at entry to retirement: no annuitization (column 4) and 50% annuitization (column 5). The comparison in all cases is relative to the benchmark economy with constant contribution rates, a "100-minus-age" equity share, and full amortization during retirement.

6.3 Comparison to other dimensions of pension reform

We have argued that redesigning contribution rates to incorporate the basic principles of consumption-saving theory has sizable welfare benefits. But how do these welfare benefits compare to changing alternative margins of a pension plan? In other words, if policy makers can only change one thing about a pension system, what should they focus on? In this section we compare our welfare benefits with the benefits of changing alternative margins of a pension system to emphasize the importance of focusing on the contribution rates.

First, we consider alternative investment strategies for the DC account. In our benchmark environment, the equity share follows the glide path "100-minus-age". To investigate the impact of changing this investment strategy we consider two alternative policies: On the one hand, we consider a fixed equity share throughout working life. The level of this equity share is set to 47%, the volume-weighted average in the benchmark economy. This is a less flexible specification than in the benchmark economy. On the other hand, we consider a rule for the equity share in the mandatory DC plan that allows for more flexibility. We implement the rule that was found to be optimal in a comparable economy by Dahlquist, Setty, and Vestman (2018). In this case, the equity share follows a rule of thumb that takes into account the age and income of an investor as well as whether this investor participates in the equity market in his non-mandated investments.

Table 10 displays the results. For comparison, column 1 repeats the welfare gains from our proposed rule for contribution rates while columns 2 and 3 show the welfare effects of changing the investment strategy. As expected, moving from an age-dependent glide path to a constant equity share leads to a welfare loss. In contrast, following a rule of thumb that mimics the optimal equity share by considering more investor heterogeneity in the form of income and stock market exposure increases welfare. However, the welfare effects of either of those changes is more than an order of magnitude smaller than the welfare effects of optimizing the rules for contribution rates.

Second, we investigate the welfare impact of redesigning the decumulation phase of the mandatory system. In our benchmark environment, the whole DC account balance is annuitized at entry to retirement. We investigate two alternative specifications of this margin: We consider one scenario where the whole balance of the DC account is paid out as a lump-sum payment at entry to retirement and can be spent flexibly by the investors. In another scenario, half of the DC account balance is annuitized, while the remaining balance is paid out as a lump-sum payment. Columns 4 and 5 of table 10 show the welfare impact of these alternative policies. As with alternative investment strategies, the welfare impact of these alternative decumulation strategies is more than an order of magnitude smaller than the gains from redesigning contribution rates. Note that there is a literature that finds high welfare gains of annuity payments (see Mitchell, Poterba, and Warshawsky (1999) for an early contribution). Our analysis differs from that literature in two main dimensions. First, our exercise is limited to varying the degree of annuitization of the DC account. In all instances, the first pillar remains unchanged and investors receive significant annuities from the notional account. Second, our welfare calculation is done at age 25, rather than 65. We do this in order to compare the benefits of two features of the second pillar: flexibility in contribution rates with the degree of annuitization.

To summarize, in this section we have shown that the recommendation to redesign contribution rates according to consumption-savings theory is robust to various sources of potential model misspecification. While the exact magnitude of the welfare benefits varies with different assumptions about investor preferences, incorporating the two basic principles of age and asset-balance dependence robustly leads to welfare improvements. Moreover, these welfare benefits turn out to be more than an order of magnitude larger than redesiging the investment or decumulation strategy of mandatory DC pension plans. This emphasizes the importance for policy makers to focus on redesigning contribution rates.

7 Concluding remarks

In this paper, we have shown that simple changes to existing DC pension plans—reflecting the principles of optimal consumption-savings theory—could substantially increase welfare while maintaining the same average replacement rate. Specifically, we have demonstrated that optimal contribution rates should be increasing with the age of the investor and decreasing with the balance-to-income ratio. Empirically, we demonstrated that in Sweden, there is large heterogeneity in the extent to which individuals incorporate these principles in their savings decisions. In particular, while individuals with sufficient liquid wealth behave in line with the predictions from theory, almost half of the population behaves hand-to-mouth and therefore saves exactly the mandated, constant fraction of income. Using a quantitative life-cycle model with a detailed pension system, we have shown that redesigning those contribution rates to instead incorporate the fundamental principles leads to substantial welfare gains without changing the average replacement rate.

Moreover, our proposed rule reduces the dispersion in the replacement rates of retirement income with respect to labor income. We stress that this reduction stems from avoiding both very low and very high replacement rates: Individuals who fall behind the target DC account balance given their income are automatically mandated to save more to avoid insufficient resources during retirement to maintain their consumption level. This avoids very low replacement rates. On the other hand, individuals who have already accumulated more than what the target balance-to-income requires will contribute less. This allows them to smooth their consumption between remaining working life and retirement and avoids excessively high replacement rates.

We have also shown that the optimality of designing contribution rates according to consumption-savings theory is robust to various sources of potential model misspecification. In particular, it is robust to a setup where investors suffer from the temptation to overconsume. Such preferences are often used as justification for the existence of mandatory pension systems. Moreover, we show that the recommendations are robust to a wide range of preference parameters. Finally, we found that the welfare benefits of redesigning contribution rates strongly exceed the benefits of reforming other aspects of pension systems such as the investment strategy and decumulation phase. Taking all our analyses together, we thus conclude that pension plan designers should take consumption-savings theory into account when setting contribution rates.

Our simple proposed rule relies only on two statistics that are readily available to the pension fund manager, namely age and the account balance. In principle, additional statistics that predict income trends could be used to refine this rule. Such statistics could be, e.g., industry, occupation, or education. Since DC pension plans in practice are often organized separately by occupation or industry, a refinement along those dimensions would be a natural extension. In that case, the welfare benefits and reduction in the dispersion of replacement rates in this paper can be seen as a lower bound for the potential benefits of aligning contribution rates with the principles of consumption-savings theory.

Finally, our analysis focused on the design of contribution rates in a mandatory DC pension plan. Nevertheless, the insights of our analysis are more widely applicable. For example, as part of the Fintech evolution, the importance of software-based, algorithmic financial advice ("robo advising") is steadily increasing. The simple rules for contribution rates that we derived in this paper could guide the advice of such services.

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Appendix

Designing Pension Plans According to Consumption-Savings Theory

A Proofs of propositions in section 2

To solve the model we reformulate the optimization problem recursively. In period $t = 3$ the individual does not make any decisions and instead simply consumes all available resources:

$$C_{i,3} = A_{i,2}R_2. \tag{35}$$

We can thus define the value function in period $t = 3$, V_3, as

$$V_3(A_{i,2}R_2) = \log(A_{i,2}R_2). \tag{36}$$

Middle-aged individuals in period $t = 2$ anticipate this and solve the following optimization problem:

$$V_2(A_{i,1}R_1, Y_{i,2}) = \max_{C_{i,2}, A_{i,2}} \log(C_{i,2}) + \beta V_3(A_{i,2}R_2) \tag{37}$$

$$\text{s.t.} \qquad C_{i,2} = A_{i,1} \cdot R_1 + Y_{i,2} - A_{i,2}, \tag{38}$$

where V_2 denotes the value function in period $t = 2$. Finally, young individuals anticipate the optimal behavior later in life and solve the following maximization problem:

$$V_1(Y_{i,1}) = \max_{C_{i,1}, A_{i,1}} \log(C_{i,1}) + \beta V_2(A_{i,1}R_1, Y_{i,2}) \tag{39}$$

$$\text{s.t.} \qquad C_{i,1} = Y_{i,1} - A_{i,1}. \tag{40}$$

Proof of proposition 2 To solve the optimization problem of the middle-aged individual ($t = 2$) we reformulate the optimization problem in equations (37) and (38) to:

$$V_2(A_{i,1}R_1, Y_{i,2}) = \max_{A_{i,2}} \log(A_{i,1}R_1 + Y_{i,2} - A_{i,2}) + \beta \log(A_{i,2}R_2) \tag{41}$$

This implies the following first-order condition:

$$-\frac{1}{A_{i,1}R_1 + Y_{i,2} - A_{i,2}} + \beta\frac{1}{A_{i,2}} = 0. \tag{42}$$

Solving for the optimal savings $A_{i,2}$ leads to

$$A_{i,2} = \frac{\beta}{1+\beta}(A_{i,1}R_1 + Y_{i,2}). \tag{43}$$

Substituting into the definition of the contribution rate (equation (6)) results in the optimal contribution rate in $t = 2$ and the optimal reactions to shocks to the balance-to-income:

$$\lambda_2^* = \frac{\beta}{1+\beta} - \frac{1}{1+\beta}\frac{A_{i,1}R_1}{Y_{i,2}} \tag{44}$$
$$\frac{\partial \lambda_2^*}{\partial \frac{A_{i,1}R_1}{Y_{i,2}}} = -\frac{1}{1+\beta} < 0.$$

Proof of proposition 1 Substituting equations (38) and (43) and the constraint (40) into the objective function in period $t = 1$ leads to the optimization problem:

$$\max_{A_{i,1}} \quad \log(Y_{i,1} - A_{i,1}) + \beta\left(\log\left(\frac{1}{1+\beta}(A_{i,1}R_1 + Y_{i,2})\right) + \beta\log\left(\frac{\beta}{1+\beta}(A_{i,1}R_1 + Y_{i,2})R_2\right)\right). \tag{45}$$

This implies the following first-order condition:

$$-\frac{1}{Y_{i,1} - A_{i,1}} + \beta\frac{R_1}{A_{i,1}\cdot R_1 + Y_{i,2}} + \beta^2\frac{R_1}{A_{i,1}\cdot R_1 + Y_{i,2}} = 0. \tag{46}$$

Solving for optimal savings in $t = 1$ leads to

$$A_{i,1} = \frac{\beta+\beta^2}{1+\beta+\beta^2}Y_{i,1} - \frac{1}{1+\beta+\beta^2}\frac{Y_{i,2}}{R_1}. \tag{47}$$

Substituting into the definition of the contribution rate (equation (5)) results in the optimal contribution rate and reaction to changes in income growth in $t = 1$:

$$\begin{aligned} \lambda_1^* &= \frac{\beta + \beta^2}{1 + \beta + \beta^2} - \frac{1}{1 + \beta + \beta^2} \frac{Y_{i,2}}{R_1 Y_{i,1}} \\ \frac{\partial \lambda_1^*}{\partial \frac{Y_{i,2}}{Y_{i,1}}} &= -\frac{1}{(1 + \beta + \beta^2) R_1} < 0. \end{aligned}$$

Proof of proposition 3 Substituting the optimal savings in period $t = 1$ (equation (47)) into the optimal contribution rate in period $t = 2$ (equation (8)) leads to the optimal contribution rate in period $t = 2$ as a function of income growth of

$$\lambda_2^* = \frac{1 + \beta^2}{1 + \beta + \beta^2} - \frac{\beta}{(1 + \beta + \beta^2)} \frac{Y_{i,1} R_1}{Y_{i,2}}. \tag{48}$$

Equating the optimal contribution rates in periods $t = 1$ and $t = 2$ we obtain

$$\begin{aligned} \frac{(\beta + \beta^2)}{(1 + \beta + \beta^2)} - \frac{1}{(1 + \beta + \beta^2)} \frac{Y_{i,2}}{R_1 Y_{i,1}} &= \frac{1 + \beta^2}{1 + \beta + \beta^2} - \frac{\beta}{(1 + \beta + \beta^2)} \frac{Y_{i,1} R_1}{Y_{i,2}} \\ \Leftrightarrow \quad 0 &= \beta \left(\frac{Y_{i,1} R_1}{Y_{i,2}} \right)^2 - (1 - \beta) \frac{Y_{i,1} R_1}{Y_{i,2}} - 1. \end{aligned}$$

Solving for $\frac{Y_{i,1} R_1}{Y_{i,2}}$ we obtain

$$\frac{Y_{i,1} R_1}{Y_{i,2}} = \frac{(1 - \beta) + \sqrt{(1 - \beta)^2 + 4\beta}}{2\beta}, \tag{49}$$

where the second solution of the quadratic equation was dropped since it is negative. This leads to the following solution for the value of income growth at which the optimal contribution rates are constant throughout working life:

$$\frac{Y_{i,1}}{Y_{i,2}} = \kappa(R_1, \beta) \quad \text{where}$$
$$\kappa(R_1, \beta) = \frac{(1-\beta) + \sqrt{(1-\beta)^2 + 4\beta}}{2\beta R_1}.$$

B Details on the empirical analysis

Table A.1: Sampling restrictions

	Type of restriction	Observations	Unique individuals	Age	Disposable income	Net worth	Stock market participation
0.	Full sample	33 802 670	5 618 949	44	172 216	360 958	.57
1.	Excl. individuals who do not reside in Sweden	33 446 485	5 575 267	44	173 497	362 709	.58
2.	Excl. individuals who are farmers or entrepreneurs	32 049 447	5 470 284	44	173 965	354 502	.57
3.	Excl. individuals who hold financial derivatives in time t or $t-1$	31 889 583	5 466 005	44	173 821	344 002	.57
4.	Excl. individuals who hold stocks or mutuals funds with missing prices or ISIN's in time t or $t-1$	29 338 880	5 320 851	44	169 862	269 031	.54
5.	Excl. individuals who have extreme financial portfolio returns in a given year (top and bottom the 1 percent)	29 043 107	5 316 200	44	169 684	269 071	.54
6.	Excl. individuals who have big changes in net worth (top and bottom 2.5 percent)	27 500 060	5 720 190	44	168 106	230 943	.53
7.	Excl. individuals who own commercial real estate	25 578 993	4 954 295	44	169 473	229 945	.51
8.	Excl. individuals who have gross labor income below the Price Base Amount (plus 5 percent) of time t	20 357 388	4 225 706	43	188 297	253 985	.56
9.	Excl. individuals who have zero or negative disposable income	20 345 737	4 224 153	43	188 475	230 011	.56
10.	Excl. individuals who are not in the sample in t, $t-1$ and $t-2$	16 911 101	3 629 474	43	191 503	262 535	.57
11.	Excl. individuals who have extreme contribution rates relative to their age cohort (top and bottom 1 percent).	16 134 639	3 509 615	43	193 873	247 485	.56

Note: The raw data from Statistics Sweden contains 73 202 337 observations from 10 152 351 unique individuals. Our full sample, in the first row, excludes individuals aged below 25 and above 64, and year 2000 from the raw data delivered by Statistics Sweden. The stock market participation rate is reported in decimals.

Table A.2: First-stage IV estimates of savings rates and asset balance-to-income ratios

	(1)	(2)	(3)	(4)	(5)
$\frac{A_{it-1}}{Y_{it}^{Disp}} \times \overline{R_{it}^{A}}$	0.973***	0.97***	0.966***	0.962***	0.976***
	(0.004)	(0.004)	(0.005)	(0.003)	(0.007)
Age Group	25-64	25-35	36-45	46-55	56-64
Controls	Yes	Yes	Yes	Yes	Yes
Individual FE	Yes	Yes	Yes	Yes	Yes
Year FE	Yes	Yes	Yes	Yes	Yes
R^2	0.996	0.994	0.996	0.996	0.997
Observations	8,872,650	2,212,862	2,282,590	2,178,616	1,820,876

Note: The table reports the first-stage estimates corresponding to Table 3. The control variables are Y_{it}^{Disp}, NW_{it-1} and $ND_{it,t-1}$. Standard errors, clustered at the level of the individual and the individual's largest security holding, are in parentheses. The individual's largest security holding is the particular stock or mutual fund—identified by their International Securities Identification Number (ISIN)—with the largest weight in the individual's financial asset portfolio. When the largest holding is bonds, bank accounts or capital insurance accounts, we classify the largest security holding by their respective asset type. Singleton groups are excluded. $* = p < 0.10$, $** = p < 0.05$, $*** = p < 0.01$. Dependent variable: $\frac{A_{it-1}}{Y_{it}^{\text{Disp}}} \times R_{it}^{A}$.

Table A.3 reports estimates from the following regression on savings amounts:

$$\Delta\widetilde{A}_{it} = \theta_i + \delta_t + \beta_1 A_{it-1} \times R^A_{it} + \beta_2 Y^{\text{Disp}}_{it} + \beta_3 NW_{it-1} + \beta_4 ND_{it,t-1} + \varepsilon_{it}. \tag{50}$$

Table A.4 reports the first-stage estimates corresponding to columns 3-7.

Table A.3: Response in savings cash-flows to changes in financial wealth

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
	OLS		IV				
$A_{it-1} \times R^A_{it}$	-0.179***	-0.196***	-0.197***	-0.238***	-0.235***	-0.231***	-0.236***
	(0.023)	(0.023)	(0.024)	(0.023)	(0.028)	(0.029)	(0.026)
Age Group	25-64	25-64	25-64	25-35	36-45	46-55	56-64
Controls	No	Yes	Yes	Yes	Yes	Yes	Yes
Individual FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Year FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes
First-stage *F*			$19,955$***	$77,657$***	$72,329$***	$65,234$***	$1,817$***
R^2	0.339	0.342					
Observations	8,900,223	8,872,650	8,872,650	2,212,862	2,282,590	2,178,616	1,820,876

Note: The sample is restricted to stock market participants (i.e., individuals with non-zero stock, mutual fund or capital insurance holdings). The control variables are Y^{Disp}_{it}, NW_{it-1} and $ND_{it,t-1}$. Standard errors, clustered at the level of the individual and the individual's largest security holding, are in parentheses. The individual's largest security holding is the particular stock or mutual fund—identified by their International Securities Identification Number (ISIN)—with the largest weight in the individual's financial asset portfolio. When the largest holding is bonds, bank accounts or capital insurance accounts, we classify the largest security holding by its respective asset type. Singleton groups are excluded. Table A.2 reports first-stage IV estimates. $* = p < 0.10$, $** = p < 0.05$, $*** = p < 0.01$.

Table A.4: Response in savings cash-flows to changes in financial wealth - first stage

	(1)	(2)	(3)	(4)	(5)
$A_{it-1} \times \bar{R}_{it}^{A}$	-0.187^{***}	-0.225^{***}	-0.221^{***}	-0.218^{***}	-0.219^{***}
	(0.023)	(0.021)	(0.026)	(0.027)	(0.026)
Age Group	25-64	25-35	36-45	46-55	56-64
Controls	Yes	Yes	Yes	Yes	Yes
Individual FE	Yes	Yes	Yes	Yes	Yes
Year FE	Yes	Yes	Yes	Yes	Yes
R^2	0.339	0.380	0.372	0.374	0.392
Observations	8,872,650	2,212,862	2,282,590	2,178,616	1,820,876

Note: The table reports the first-stage estimates corresponding to Columns 3-7 in Table A.3. Dependent variable: $A_{it-1} \times R_{it}^{A}$. $* = p < 0.10$, $** = p < 0.05$, $*** = p < 0.01$.

Table A.5 reports the first-stage estimate corresponding to Column 2, 4, and 6 of Table 4. We instrument income growth with the earnings growth of the individuals' employer. The estimation is based on a subsample of individuals that have the same employer in two consecutive years. The subsample contains 220,936 employers, and the average employee-to-employer ratio per year is: 14.9 (2002), 15.5 (2003), 15.6 (2004), 15.3 (2005), 15.4 (2006) and 15.2 (2007).

Table A.5: First-stage IV estimates for income shocks

	(1) Full Sample	(2) Hand-to-Mouth	(3) Non-Hand-to-Mouth
$w_{i,t}^{Employer} - w_{i,t-1}^{Employer}$	0.103***	0.107***	0.093***
	(0.001)	(0.001)	(0.002)
Individual FE	Yes	Yes	Yes
Year FE	Yes	Yes	Yes
Individual Clusters	Yes	Yes	Yes
Employer Clusters	Yes	Yes	Yes
R^2	0.283	0.052	0.044
Observations	12,121,447	5,626,553	6,785,791

Notes: The table reports the first-stage estimates for Columns (2), (4) and (6) of Table 4. The variable $w_{i,t}^{Employer} - w_{i,t-1}^{Employer}$ is the aggregate wage bill growth of individual i's employer. $* = p < 0.10$, $** = p < 0.05$, $* * * = p < 0.01$.

C The Swedish pension system

The Swedish pension system rests on three pillars: public pensions, occupational pensions, and private savings. Below, we describe the public and occupational pensions.

The public pension system was reformed in 2000.[31] It has two major components referred to as the income-based pension and the premium pension. A means-tested benefit provides a minimum guaranteed pension.

The contribution to the income-based pension is 16% of an individual's income, though the income is capped (in 2014, the cap was SEK 426,750, or approximately USD 62,200). The return on the contribution equals the growth rate of aggregate labor income measured by an official "income index". Effectively, the return on the income-based pension is similar to that of a real bond. The income-based pension is notional in that it is not reserved for the individual but is instead used to fund current pension payments as in a traditional pay-as-you-go system. It is worth mentioning that the notional income-based pension is also DC, but to avoid confusion we simply refer to it as the notional pension.

The contribution to the premium pension is 2.5% of an individual's income (capped as above). Unlike the income-based pension, the premium pension is a fully funded DC account used to finance the individual's future pension. Individuals can choose to allocate their contributions to up to five mutual funds from a menu of several hundred. The premium pension makes it possible for individuals to gain equity exposure. Indeed, most of the investments in the system have been in equity funds (see, e.g., Dahlquist et al., 2015). A government agency manages a default fund for individuals who do not make an investment choice. Up to 2010, the default fund invested mainly in stocks but also in bonds and alternatives. In 2010, the default fund became a life-cycle fund. At the time of retirement, the savings in the income-based pension and the premium pension are transformed into actuarially fair life-long annuities.

In addition to public pensions, approximately 90% of the Swedish workforce is entitled to

[31] Individuals born between 1938 and 1954 are enrolled in a mix of the old and new pension systems, while individuals born after 1954 are enrolled entirely in the new system.

occupational pensions. Agreements between labor unions and employer organizations are broad and inclusive and have gradually been harmonized across educational and occupational groups. For individuals born after 1980, the rules are fairly homogeneous, regardless of education and occupation. The contribution is 4.5% of an individual's income (capped as above) and goes into a designated individual DC account. For the part of the income that exceeds the cap, the contribution rate is greater in order to achieve a high replacement rate even for high-income individuals. While the occupational pension is somewhat more complex and tailored to specific needs, it shares many features with the premium pension. Specifically, it is an individual DC account.

D Properties of contribution rates

The functional form assumption of the contribution rates in (31) leads to the following semi-elasticities of the contribution rate to changes in income, DC account balance, or balance-to-income ratio:

$$\frac{\partial \lambda_{it}}{\partial Y_{it}} \cdot Y_{it} = -b_2 \frac{\frac{B_{it}}{Y_{it}}}{\chi_t}, \tag{51}$$

$$\frac{\partial \lambda_{it}}{\partial B_{it}} B_{it} = b_2 \frac{\frac{B_{it}}{Y_{it}}}{\chi_t}, \tag{52}$$

$$\frac{\partial \lambda_{it}}{\partial \frac{B_{it}}{Y_{it}}} \frac{B_{it}}{Y_{it}} = b_2 \frac{\frac{B_{it}}{Y_{it}}}{\chi_t}. \tag{53}$$

Thus, for an investor who has a balance-to-income ratio that is right on target, $\frac{B_{it}}{Y_{it}} = \chi_t$, the contribution rate will change by b_2 for a percentage increase in B_{it} or $\frac{B_{it}}{Y_{it}}$ or for a percentage decrease in Y_{it}.

Moreover, the functional form has implications for the elasticity of disposable income ($Y_{it}^{disp} = (1 - \lambda_{it})Y_{it}$) to changes in gross income (Y_{it}):

$$\begin{aligned} \frac{\partial Y_{it}^{disp}}{\partial Y_{it}} \cdot \frac{Y_{it}}{Y_{it}^{disp}} &= \left(-\frac{\partial \lambda_{it}}{\partial Y_{it}} Y_{it} + (1 - \lambda_{it}) \right) \cdot \frac{Y_{it}}{Y_{it}^{disp}} \\ &= \left(1 - \lambda_t + b_2 \frac{\frac{B_{it}}{Y_{it}}}{\chi_t} \right) \frac{Y_{it}}{(1 - \lambda_{it}) Y_{it}} \\ &= 1 + \frac{b_2 \frac{\frac{B_{it}}{Y_{it}}}{\chi_t}}{1 - \lambda_{it}}. \end{aligned} \tag{54}$$

Inserting the functional form of the contribution rate (31), the elasticity of disposable income to changes in gross income is positive if and only if

$$
\begin{aligned}
1+\frac{b_2\frac{\frac{B_{it}}{Y_{it}}}{\chi_t}}{1-\lambda_{it}} &\geqslant 0 \\
-b_2\frac{\frac{B_{it}}{Y_{it}}}{\chi_t} &\leqslant 1-b_0-b_1t-b_2\left(\frac{\frac{B_{it}}{Y_{it}}}{\chi_t}-1\right) \\
b_0+b_1t-b_2 \leqslant 1. & \qquad (55)
\end{aligned}
$$

E Algorithm to find optimal policies for the contribution rate

To determine the optimal policy for contribution rates we select the design that delivers the highest welfare gain while achieving the same average replacement rate out of the DC account as the benchmark constant contribution rate.

For each replacement-rate target we proceed in four steps:

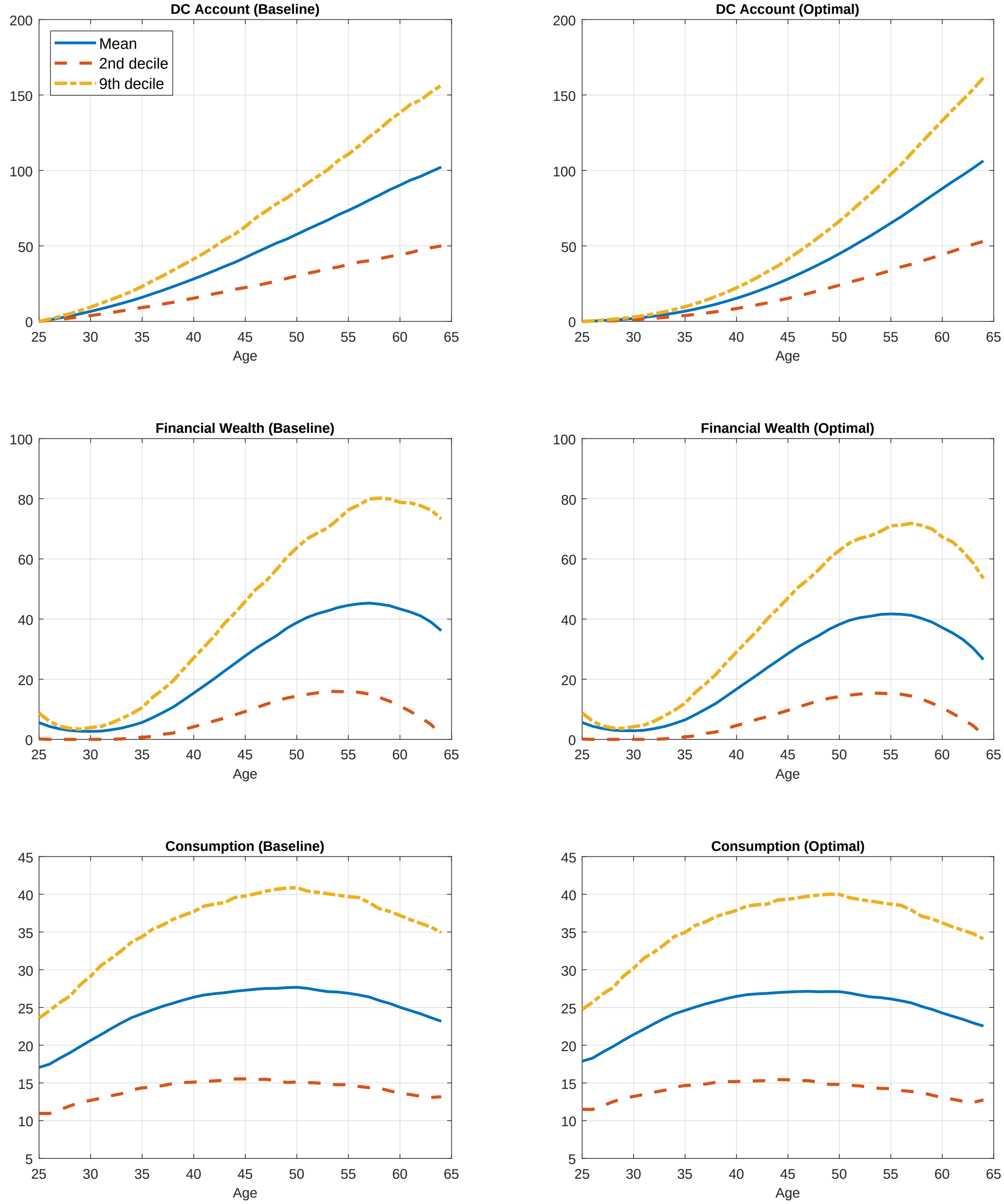
1. solve for the constant contribution rate that delivers this average replacement rate
2. search for the optimal policy that allows for an age dependency in the policy ($\lambda_{it} = \beta_0 + \beta_1 t$):
   (a) for each candidate coefficient β_1 solve for the required constant β_0 such that the policy achieves exactly the required average replacement rate
   (b) select the design (i.e. candidate coefficient) with the highest welfare gain
3. search for the optimal policy that allows for a dependency on the balance-to-income ratio ($\lambda_{it} = \beta_0 + \beta_2 \left(\frac{\frac{B_{it}}{Y_{it}}}{\chi_t} - 1 \right)$):
   (a) for each candidate for the coefficient β_2, solve for the required constant β_0 and vector of target balance-to-income ratios χ_t such that
      - the design achieves exactly the required average replacement rate and
      - the average contribution rate is constant for all ages
   (b) select the design (i.e. candidate coefficient) with the highest welfare gain
4. search for the optimal policy that allows for dependencies on both age and balance-to-income ratio ($\lambda_{it} = \beta_0 + \beta_1 t + \beta_2 \left(\frac{\frac{B_{it}}{Y_{it}}}{\chi_t} - 1 \right)$):
   (a) for each combination of candidates for the coefficient of β_1 and β_2, solve for the required constant β_0 and vector of target balance-to-income ratios χ_t such that
      - the design achieves exactly the required average replacement rate and

- the average contribution rate follows exactly the age trend $\beta_0 + \beta_1 t$

(b) select the design (i.e. combination of coefficients) with the highest welfare gain

F Additional results about flexible contribution rates

Figure A.1: Benchmark vs. flexible pension system



Note: The left columns show the distributions for the DC account balance, financial wealth and consumption (the average and the 2nd and 9th deciles) for the benchmark calibrations. The right columns show the corresponding distributions for the optimal flexible design. Values are expressed in SEK 10,000s.

Table A.6: Effect of the age-only design on behavior

	Age 30	Age 40	Age 50	Age 60
Panel A: Average Consumption				
Benchmark	20.6	26.4	27.7	25.0
Optimal	21.3	26.2	27.4	24.7
Changes (percent)	3.3	-0.6	-1.1	-1.4
Panel B: Average Financial Wealth				
Benchmark	2.7	15.4	38.8	43.4
Optimal	3.2	20.9	45.9	43.3
Changes (percent)	19.3	35.9	18.3	-0.0
Panel C: Average Stock Market Participation				
Benchmark	0.32	0.55	0.55	0.55
Optimal	0.42	0.72	0.72	0.72
Changes (percent)	30.5	31.4	31.0	31.0

Note: The table reports average consumption, financial wealth, and participation at different points in the life cycle. It compares the levels in the benchmark calibration and under the age-dependent design of contribution rates (in SEK 10,000s) and computes the change from benchmark to age-dependent in percent.

Table A.7: Effect of the B/Y-only design on behavior

	Age 30	Age 40	Age 50	Age 60
Panel A: Average Consumption				
Benchmark	20.6	26.4	27.7	25.0
Optimal	20.7	26.7	27.5	24.7
Changes (percent)	0.3	1.3	-0.7	-1.2
Panel B: Average Financial Wealth				
Benchmark	2.7	15.4	38.8	43.4
Optimal	2.3	11.2	31.1	36.3
Changes (percent)	-14.1	-27.5	-19.9	-16.4
Panel C: Average Stock Market Participation				
Benchmark	0.32	0.55	0.55	0.55
Optimal	0.23	0.25	0.25	0.25
Changes (percent)	-30.4	-54.5	-54.8	-54.8

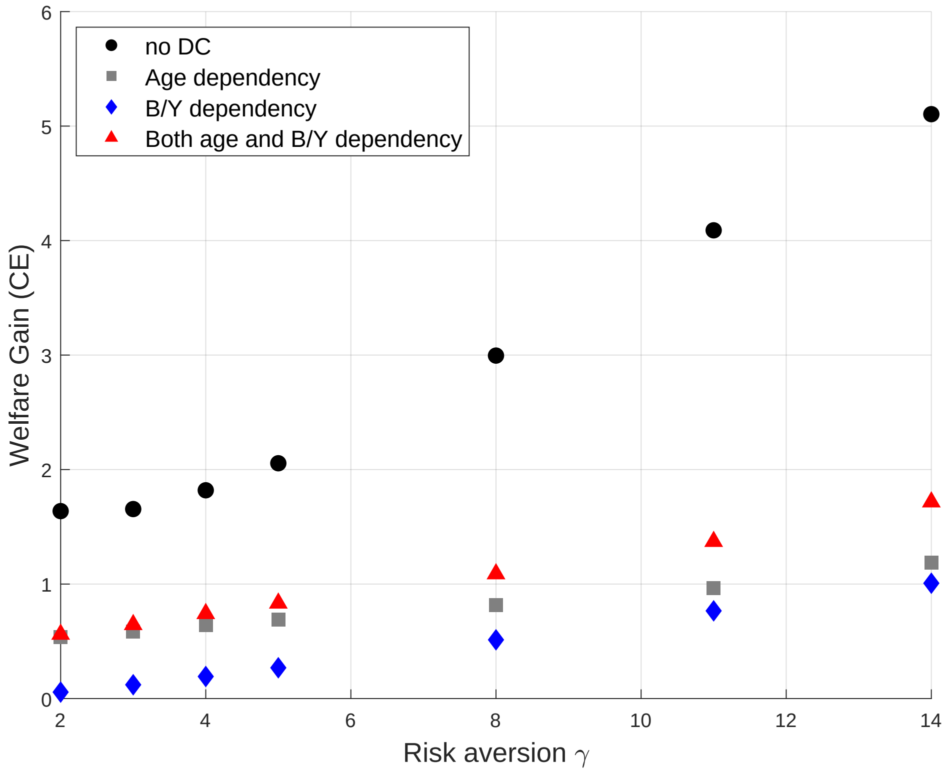
Note: The table reports average consumption, financial wealth, and participation at different points in the life cycle. It compares the levels in the benchmark calibration and under the B/Y-dependent design of contribution rates (in SEK 10,000s) and computes the change from benchmark to B/Y-dependent in percent.

Table A.8: Changes in disposable income—standard deviation and correlation with shocks

	Standard Deviation	Correlation with Changes in Gross Income	Correlation with Stock Market Returns
Benchmark	21.24	1.00	-0.0014
Age only	20.98	1.00	-0.0013
BY only	19.06	0.94	-0.0059
Optimal	19.26	0.95	-0.0074

Note: The table reports statistics of changes in disposable income: average lifetime standard deviation, correlation with changes in gross income, and correlation with changes in stock market returns.

Figure A.2: Risk aversion and welfare gains - no longevity insurance



Note: The figure shows the welfare gains (expressed as income compensation) of the alternative designs of contribution rates for different degrees of temptation. Note that, for each degree of temptation, the model has been recalibrated to match data moments (hence, preference parameters β and $\bar{\kappa}$ vary across different horizons).