

The Design of ‘Soft’ Welfare-to-Work Programs*

Nicola Pavoni[†] *Ofer Setty*[‡] and *Giovanni L. Violante*[§]

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Abstract

This paper models welfare-to-work programs as contracts offered by the principal/government to unemployed agents in an environment with moral hazard. A welfare-to-work program comprises of several policy instruments (e.g., job-search, assisted search, mandated work) the principal can use, in combination with welfare benefits, in order to minimize the costs of delivering promised utility to the agent. The generosity of the program and the skill level of the unemployed agent determine the optimal policy instrument to be implemented. Restricting attention to ‘soft programs’ —contracts that make no use of punishments or sanctions— allows a fully analytical characterization of the optimal program and, in addition, it makes the solution robust to hidden saving.

Keywords: Job-search assistance, Hidden saving, Human capital, Social assistance, Unemployment insurance, Welfare-to-Work, Work requirements.

JEL Classification: D82, H21, J24, J64, J65.

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[†]Bocconi University, CEPR, and IFS

[‡]Tel Aviv University

[§]New York University, CEPR, IFS, and NBER.

1 Introduction

Welfare-to-work programs are government programs aimed at the poor and the jobless. Their objective is providing income support and, at the same time, promoting economic self-sufficiency through work. Achieving this objective is challenging because the provision of assistance interferes with individual incentives to exert effort to find and retain a suitable job. In order to tackle successfully this insurance-incentive trade-off, governments use a wide range of policy instruments.

The backbone of most welfare programs around the world is a combination of Unemployment Insurance and Social Assistance. We use the term Social Assistance as an umbrella to capture all those policies of pure income support such as the Supplemental Nutritional Assistance Program (SNAPS, formerly called Food Stamps), child-care and housing assistance, or Disability Insurance in the case of the United States.

A large class of policies goes under the heading of Job Search Assistance. Here, we focus only on those interventions where the public employment agency actively helps the participant to find work by providing job contacts with suitable employers.¹ A variety of interventions are based on work requirements. At one end of the spectrum, the work requirement is purely intended as a social obligation for the receipt of the welfare check. At the opposite end, the work requirement is meant to function as a transition into self-sufficiency through private employment. For example, while the participant is mandated to work in a public or non-profit agency, the caseworker actively assists her search for private employment in a similar job. Or, the caseworker directly matches the individual to a private employer with the expectation that she might be retained by that same employer. To distinguish the first type of work (work in exchange for benefits) from the second (stepping stone to private employment), we label them Mandatory Work and Transitional Work.² Finally, many programs include financial incentives to employment such as re-employment bonuses and earnings subsidies, for example the U.S.

¹To enforce active job search, some programs require the welfare recipient to show evidence of her job-search efforts (applications, contacts, interviews) to the caseworker. The optimal use of monitoring of search activities is analyzed extensively in [Pavoni and Violante \(2007\)](#), and [Setty \(2014\)](#). Other search-based interventions focus on the development of job-seeking and interviewing skills. [Wunsch \(2013\)](#) studies the optimal design of this type of intervention.

²Another important class of interventions based on training is aimed at augmenting the market value of skills of the participant. We abstract from these interventions in this paper, and return to this point in the Conclusions, where we discuss directions to extend our analysis.

Fraction of welfare (TANF) recipients				
State	Inactive	Search	Work	Training
Illinois	0.43	0.03	0.42	0.12
Pennsylvania	0.59	0.08	0.10	0.23
Michigan	0.72	0.18	0.04	0.06
New York	0.82	0.04	0.09	0.05

Table 1: Use of policy instruments across U.S. states. Source: Department of HHS. Year 2009.

Earned Income Tax Credit.

The United States is an interesting case to study because the 1996 Personal Responsibility and Work Opportunity Reconciliation Act (PRWORA) deeply reformed the system of cash welfare assistance for poor households. In particular, it removed federal regulatory power over the structure of these programs, and gave states full flexibility in choosing policy instruments. As a result, a range of diverse programs was implemented across US states over time: some programs are more focused on assisting individual job search, others on education and training, others on moving the individual as soon as possible into some form of work.

Table 1 illustrates some key characteristics of welfare-to-work programs for four large US states: Illinois, Pennsylvania, Michigan, and New York. It reports the fraction of recipients of the Temporary Assistance for Needy Families (TANF) —the main form of welfare benefits— not employed in the private sector, by activities they are required to perform.³ The first fact that emerges is the large dispersion in the fraction of inactive individuals, those exempted from any activity, among these four states. The second fact is that different states emphasize different activities. In Michigan, the most common required activity is job-search, in Pennsylvania training, and in Illinois mandated work. In conclusion, it appears that states take very heterogeneous approaches to the design of welfare-to-work programs. A first natural question is therefore: what explains this heterogeneity?

The 1996 welfare reform also allowed states to set their own TANF benefit level and, once again, this freedom of choice led to significant heterogeneity across states. Figure 1 illustrates this point by grouping states by the maximum monthly amount of TANF benefits payable to a single parent with two children. Not surprisingly, there is a correlation between the ‘color’

³See Appendix B for details on the calculation of these fractions.

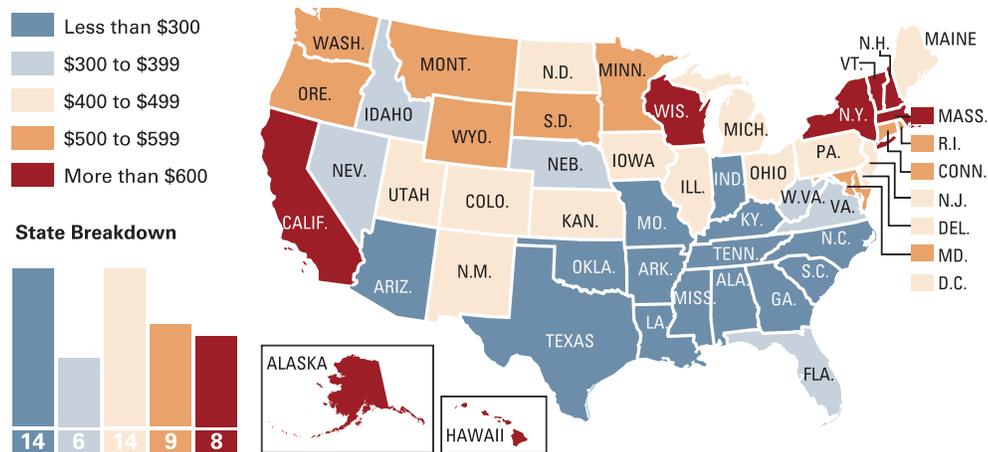


Figure 1: Maximum monthly TANF benefits for a single parent of 2 children. Source: CLASP. Year 2010

of the administration (Democratic vs. Republican) —and hence the state residents’ political preferences towards redistribution— and the degree of generosity of the state towards its welfare participants.⁴ Then, a second natural question is: given how generous a state wants to be toward its welfare participants, what mix of policy instruments should its welfare program contain? For example, in Table 1 and Figure 1, we saw that New York is one of the most generous states and, at the same time, one of the states that is most willing to tolerate inactivity from its welfare recipients. Instead, Illinois is much less generous and bases its program largely on work requirements. Are these states making the right choices in designing their welfare programs?

To address these questions, we need a normative framework to think about the optimal design of welfare programs.

Our starting point is the optimal unemployment insurance contract in the presence of a repeated moral hazard problem caused by the inability of the principal/government to observe the unemployed agent’s job search effort (Shavell and Weiss, 1979). The problem admits a recursive representation with two state variables: the expected discounted utility promised by the contract to the unemployed agent (Atkeson and Lucas, 1995; Wang and Williamson, 1996; Hopenhayn and Nicolini, 1997), and the human capital of the unemployed (Pavoni and Violante, 2007; Pavoni, 2009).

⁴In 2010, the average value of the maximum monthly TANF benefits was \$510 in Democratic states and \$390 in Republican states.

The key innovation that makes this framework amenable to analyze the optimal design of welfare programs and the trade-offs among the available policy instruments is the introduction of additional technologies, besides job search. In this paper, to model work-based and job-search assistance policies we introduce two technologies in the economic environment. First, a *secondary sector production* technology that is less productive than the (primary) one used in market-sector jobs but that, as the latter, requires effort to yield output. With these features we wish to capture the fact that work-based activities employ the welfare recipients in basic tasks with very low value added, usually in government agencies, non-profit institutions, or community service organizations. Second, an *assisted job search* technology that allows the unemployed to defer her job search to an agency at a cost. This technology frees up time from search to either work or rest.

In this framework, a policy instrument is a government's prescription of an activity—the use of a technology—to a participant, with an associated income transfer. We interpret the use of the secondary production technology alone as Mandatory Work, and the joint use of this production technology and assisted search as Transitional Work. Since the assisted search technology can always be used on its own, the model also includes a Job Search Assistance policy. In addition to these three policy instruments, the framework yields naturally Unemployment Insurance, where the worker exerts search effort on her own, and Social Assistance, corresponding to income support with no effort requirements.

The contract between the government/principal and the unemployed is the welfare-to-work program: for every level of promised utility and every skill level, the program specifies a policy instrument to be used and an amount of welfare benefits to be paid. The optimal program maximizes the participant's expected utility subject to a government budget constraint.

We restrict attention to a particular class of welfare-to-work programs that we call 'soft'. Soft welfare programs do not use any form of punishment or sanction on the participant. In essence, promised utility cannot decline along the program. This assumption is, admittedly, restrictive: it is well known, for example, that the optimal unemployment insurance contract prescribes that promised utility should fall along the unemployment spell, when the worker fails to find a job. Thus, the fully optimal dynamic contract is 'harsh'—in the sense that it comprises punishments.

There are two benefits of focusing the analysis on soft programs. First, one can obtain closed-form expressions clearly describing the trade-offs among the various policy instruments available to the government. In [Pavoni, Setty, and Violante \(2014\)](#) we show that these are also the key trade-offs in the fully optimal contract that allows punishments. Second, ‘soft’ welfare programs are robust to the hidden saving friction: even if the government does not observe the agent’s assets, the optimal program remains unchanged. It is well known, instead, that in general hidden savings disrupt incentives in a program optimally designed under the assumption that saving can be controlled by the principal ([Rogerson, 1985](#); [Kocherlakota, 2004](#); [Abraham and Pavoni, 2008](#)).

The main result of the paper is a full characterization of the optimal design of a ‘soft’ welfare-to-work (henceforth, WTW) program. In other words, we are able to determine at every point in the state space—the two dimensional space in promised utility and human capital—which policy dominates all the others. We prove that promised utility is always constant during the optimal program, and can therefore be interpreted as a parameter measuring the level of generosity of the program. Since human capital is a fixed individual characteristic, the optimal program prescribes that each worker is assigned to a single policy until she finds employment in the market sector—if the policy contemplates job search—or forever after if it does not. The choice of this policy depends on how generous the government is towards its welfare recipients and how skilled program participants are.

The rest of the paper is organized as follows. [Section 2](#) outlines the economic environment and defines the contract. [Section 3](#) discusses in detail the trade-offs between the different policy options of the program. [Section 4](#) illustrates the features of the optimal WTW program, and explains why the optimal program is robust to hidden saving. [Section 5](#) re-examines the variation in policy instruments used by welfare programs across U.S. states in light of the lessons learned from the model. [Section 6](#) concludes.

2 Economic environment

Demographics and preferences. Individuals are infinitely lived. Preferences are time-separable and the future is discounted by the factor $\beta \in (0, 1)$. Period utility over consumption c and effort a is given by $\log(c) - a$. We impose that $c \geq 0$ and $a \geq 0$. The log specification is useful

to obtain closed forms that allow us to clearly identify the forces at work in the economy, but all the key results hold with more general utility. The assumption that the disutility of effort is separable from consumption is, instead, important because it implies wealth effects on the supply of effort that drive many of the results.

Activities and effort. An individual can either rest, search for a job, or work. We assume that search and work are mutually exclusive, since they both use the whole effort endowment e .⁵ Rest corresponds to zero effort ($a = 0$), a normalization. Therefore, $a \in \{0, e\}$.⁶

Human capital. Agents are endowed with a stock of human capital (or skill level) $h \geq 0$. We assume that h is fixed—it indexes the *type* of the agent. As we discuss in detail below, the skill level affects the output of an employed worker, and the job finding probability of an unemployed worker.

Market-sector production. An agent of type h who is employed in the market sector produces output $\omega(h)$. We let $\omega(\cdot)$ be a continuous and increasing function, with $\omega(h) \in [0, \bar{\omega}]$, and $\omega(0) = 0$. Recall that production entails effort level e . Market-sector employment is an absorbing state. We therefore exclusively focus on the optimal design of a welfare program for jobless individuals.⁷

Private job search. It is useful to isolate two distinct stages in the process of searching for a market sector job: application and interview. In the first stage, an agent of type h *contacts* an employer with probability $p(h, a)$, a function that is strictly increasing in h for $a = e$, and identically equal to zero if $a = 0$. In the second stage, the firm and the agent *meet* for a job interview and draw an idiosyncratic outcome of the meeting: with probability $\theta(r)$ the worker is retained by the firm, where r is the worker's retention action. We let $r \in \{0, 1\}$ and $\theta(1) \equiv \theta > \theta(0) = 0$. The worker thus has control of the interview and can always, by choosing $r = 0$,

⁵The assumption that search and work effort are of the same magnitude is made to simplify the exposition. Recent papers based on time use data show that the unemployed devote relatively little time to search in a typical day (e.g., [Krueger and Mueller, 2012](#); [Aguiar, Hurst, and Karabarbounis, 2013](#)), but the key qualitative results of our analysis do not depend on this assumption.

⁶We can easily allow for a generalization where an agent who is resting produces b units of the consumption goods through home-production, and an agent who is working incurs additional consumption expenditures x (e.g., commuting, child care). As long as the pair (b, x) is observable, our characterization goes through.

⁷The optimal unemployment compensation contract with job separations and multiple unemployment spells is studied by [Hopenhayn and Nicolini \(2009\)](#). Their findings are relevant to our setup only in the sense that, while we assume an exogenous value for initial promised utility of the unemployed individual, with multiple spells this initial value would be endogenously determined by the entire individual employment history.

make sure that it fails and that she does not receive a job offer from the potential employer.⁸ Neither retention action entails any cost to the agent.

Combining both stages of the search technology together, the job finding probability is

$$\pi(h, a, r) = p(h, a) \cdot \theta(r),$$

where it is useful to note that if $a = 0$ or $r = 0$, then $\pi(h, a, r) = 0$ for all h . Let y denote the outcome of the search activity during unemployment, with $y \in \{f, s\}$, where f denotes ‘failure’ and s ‘success’ of job search. To simplify the notation, we denote $\pi(h, e, 1)$ as $\pi(h)$.

Job-search assistance. Upon payment of the cost κ , the job-seeker can use the services of a job-search agency that searches on behalf of the individual who, in turn, saves her job search effort. This job-search assistance technology has the same effectiveness as private search in creating a contact with a potential employer (the first stage of private search).⁹ Since the agent can always choose the retention action during the job interview (the second stage), the assisted search technology implies a conditional job finding probability $\pi(h, e, r)$, with $r \in \{0, 1\}$.

Secondary-sector production. An individual working on a secondary-sector job produces an amount of consumption goods independent of h . Secondary sector jobs are immediately available, i.e., there is no search friction. They yield a net output flow of $\underline{\omega}$.¹⁰ Also, this production technology requires work effort e . These jobs should be interpreted as a ‘make-work’ jobs: simple tasks (e.g., street and park cleaning, garbage collection, basic elderly assistance, or performing janitorial and maintenance services) that produce a minimum level of output and that are administered by a government agency, a non-profit institution, or a community service organization.

Storage, credit, and insurance. The individual starts with zero initial wealth and has access to storage where saving commands a return $R = \beta^{-1}$, but cannot access neither credit nor

⁸For example, the worker can choose to appear sloppy and uninterested about the job during the interview, or can pretend she is not competent in the required tasks.

⁹Admittedly, this is a stark version of job search assistance. One can think of alternative versions where the agency is more or less effective than individual, or where the agency input adds to the time devoted by the unemployed to job search.

¹⁰We think of $\underline{\omega}$ as the flow output produced by the worker *net* of the flow costs associated to operating this technology.

state-contingent insurance.

2.1 Principal-agent contractual relationship

We now introduce a risk-neutral planner/government who faces an intertemporal budget constraint and a real return R equal to β^{-1} . We think of the government as the principal, the unemployed as the agent, and the welfare-to-work program as the contract between these two parties.

Information structure. The use of technologies (private search, search assistance, make-work production, market-sector production) and their outcomes are observable. Human capital h is observable. Since output during market-sector work $\omega(h)$ is observable and since the technology is deterministic, work effort is also observable. However, search effort and the retention action are private information to the agent and under her full control: these are the sources of moral hazard. The assumption that search effort is unobservable is at the core of much of the literature on optimal unemployment insurance. The unobservability of the retention action boils down to the impossibility, for the planner, to force the agent to accept a market-sector job when the utility the agent derives from this choice is lower than the utility of remaining jobless. Finally, we assume that saving is observable too, but we relax this assumption in Section 4.2. Only observable variables are contractible, i.e., a contract can be made contingent only on those variables.

'Soft' programs. At time $t = 0$, the government (principal) offers the unemployed worker (agent) of type h an insurance contract (the program). The principal is able to fully commit to the contract. We use a recursive formulation of the contract with two state variables: the expected discounted continuation utility promised by the contract U , and the human capital of the worker h .¹¹

We analyze a class of contracts with the restriction that agents' expected utility *cannot decrease during unemployment*. This means that incentives can be delivered through rewards (carrot), but not through punishments (stick), hence the label 'soft' program. This restriction is implemented by a 'No Stick' constraint on the dynamics of promised utility.

¹¹The employment status is also a state, but it is trivial because the only state of interest is unemployment, and when the worker finds a job, employment is absorbing. We therefore omit it as an explicit state variable.

	Private Search/None	Search Assistance	Make-Work	Search Assistance & Make-Work
High effort	Unemployment Insurance	X	Mandatory Work	Transitional Work
Low effort	Social Assistance	Job-Search Assistance	X	X

Table 2: Policy instruments described as combinations of use of technologies and effort recommendations. Suboptimal combinations are denoted by the symbol ‘X’.

The contract specifies recommendations on: (i) use of technology, (ii) search effort and retention action, (iii) a consumption level for the agent, and (iv) continuation utility. Continuation values are contingent on the stochastic outcome of job search y , whenever private or assisted search are recommended. Note that if high search effort is recommended, the planner must also induce the high retention action, or the recommendation would be wasteful. Because the two actions are perfectly correlated, with a slight abuse of language we refer jointly to both as ‘effort.’

It is convenient to separate the components of the contract between the activity the worker is engaged in and the (current and future) payments. To this end, we define a *policy instrument*, or more simply a policy, as a combination of the first two components of the contract: (i) use of technology and (ii) effort. As summarized in Table 2, there are five, overall, possible combinations. Unemployment Insurance (UI) corresponds to the use of the private search technology alone with high effort ($a = e$); Social Assistance (SA) corresponds to ($a = 0$) and no use of any additional technology; Job Search Assistance (JA) is the combination of the job-search assistance technology and no effort ($a = 0$); Mandatory Work (MW) is the use of the make-work production technology with the high effort; finally, Transitional Work (TW) corresponds to the combined use of make-work with assisted job-search, and high effort, necessary for production.¹²

Let $i(U, h) = \{UI, SA, JA, MW, TW\}$ index the policy instrument specified by the contract at a pair (U, h) . Conditional on the policy choice, the contract also specifies a consumption level for the agent $c^i(U, h)$. During the unemployment spell, the consumption level corresponds to the unemployment compensation; during private employment, the difference between the wage level and the consumption level $\tau^i(U, h) = \omega(h) - c^i(U, h)$ implies a wage tax, or a

¹²Clearly, UI, JA, and TW also require a positive retention action ($r = 1$) recommendation.

subsidy if negative. Finally, for $i \in \{UI, JA, TW\}$, the contract specifies a continuation utility $U_i^y(U, h)$ conditional on the random, but observable, outcome of job search y .

In sum, a contract is a triplet of incentive compatible functions $\{i(U, h), c^i(U, h), U^i(U, h)\}$ where, in turn, each policy i specifies a level of search and retention effort. A contract is a *welfare-to-work program*, as it specifies for every type of worker (U, h) an activity, through the choice of policy, a payment during the activity, and a tax/subsidy conditional on finding a job in the market sector, through the choice of a continuation utility.

An optimal welfare-to-work program maximizes the expected discounted stream of net revenues for the government and guarantees the agent an expected discounted utility level U_0 . This initial condition should be thought of as an exogenous parameter measuring the generosity of the welfare system—for example, the outcome of voting or a political process.

3 The value of the policy instruments

In this section, we describe the problem solved by the planner for each one of the five policies introduced above. In what follows, we'll make use of the value function $V(U, h) = \max_i V^i(U, h)$, the upper envelope of the policy-specific values.¹³

Social Assistance. We begin with the value of SA to the planner.

$$\begin{aligned}
 V^{SA}(U, h) &= \max_{c, U^f} -c + \beta V(U^f, h) \\
 &\quad s.t. \\
 U &= \log(c) + \beta U^f, & \text{(PK)} \\
 U^f &\geq U, & \text{(NS)}
 \end{aligned}$$

Because job search is not required by the principal under this policy, there is no chance of finding a job. The PK constraint is the promise-keeping constraint, and the NS constraint represents the no-stick constraint. In Appendix A, we show that, if at (U, h) SA is the optimal choice, then

¹³ In Appendix A, we show that the principal may want to use lotteries to convexify the upper envelope, but only at time $t = 0$, depending on the initial condition U_0 . In the main text, to simplify the exposition, we proceed under the assumption that lotteries are not used. The Appendix contains a full derivation of the closed form in points where randomizations are used.

we have $U^f = U$. Therefore, if SA was optimal at (U, h) , it will remain optimal forever after, and $V = V^{SA}$. SA is therefore an absorbing state. Using this result in the PK constraint gives the expression for consumption $c^{SA}(U) = \exp((1 - \beta)U)$ which, substituted into the value function, yields

$$V^{SA}(U, h) = \frac{1}{1 - \beta} \left[A^{SA}(h) - B^{SA}(h) \cdot \exp((1 - \beta)U) \right], \quad (1)$$

where the two coefficients are

$$A^{SA}(h) = 0 \quad \text{and} \quad B^{SA}(h) = 1.$$

As we show in the rest of this section, every value associated to a policy i can be expressed like equation (1) as the sum of a policy-specific coefficient independent of U and that we call $A^i(h)$, and a term that is a function of U and can always be written as another policy-specific coefficient $B^i(h)$ times $\exp((1 - \beta)U)$.

The coefficient $A^i(h)$ has the interpretation of the per-period net return of the policy for the planner, i.e., the return net of the costs of using the assisted search or secondary production technologies. In SA, A^{SA} is zero because there is no output from production during SA, and no direct costs of the policy since no costly technology is being used. The second term $B^i(h) \cdot \exp((1 - \beta)U)$ has the interpretation of the per-period consumption-equivalent cost of delivering U units of promised utility to the agent. In SA, this cost captures only the fact that a higher promised utility entails higher consumption payments every period, so $B^{SA}(h)$ equals 1. This basis cost is, obviously, common to all policies. Since the value of SA is independent of h , in what follows we will drop the dependence on h and denote it as $V^{SA}(U)$.

Market-sector employment. We now present the value of employment in the market sector, which is useful to, later, characterize the value of UI . For an employed worker, the planner

solves:

$$W(U, h) = \max_{c, U^s} \omega(h) - c + \beta W(U^s, h) \\ \text{s.t.}$$

$$U = \log(c) - e + \beta U^s, \quad (\text{PK})$$

$$U^s \geq U. \quad (\text{NS})$$

Since employment is absorbing and features no incentive problem, the choice of continuation utility delivers $U^s = U$. Solving for consumption from the PK constraint yields

$$c^W(U) = \exp((1 - \beta)U + e). \quad (2)$$

Substituting this expression into the value function W allows us to rewrite the value of employment as:

$$W(U, h) = \frac{1}{1 - \beta} \left[A^W(h) - B^W(h) \exp((1 - \beta)U) \right], \quad (3)$$

where the two coefficients are

$$A^W(h) = \omega(h), \quad \text{and}$$

$$B^W(h) = \exp(e).$$

A^W equals $\omega(h)$ because during market employment there is a production flow every period. The coefficient B^W should be interpreted as the cost of delivering a unit of promised utility, in consumption-equivalent terms, during employment *relative to SA*. This relative cost is $\exp(e) > 1$. The reason is that production requires work effort which, in turn, reduces the agent's utility. The planner must therefore pay a higher consumption to the employed worker, compared to that paid to the agent in *SA*, in order to deliver the same level of promised utility U .

We call this additional cost the *effort compensation cost* and note —by comparing $c^{SA}(U)$ and $c^W(U)$ — that this higher cost manifests itself as higher consumption payments relative to *SA*. The fact that the effort compensation cost enters multiplicatively with respect to promised utility indicates a negative wealth effect that is crucial for some of our results: since $u = \log$ is a

strictly concave function, and effort enters additively into agent's utility, the higher the average consumption promised by the principal to the agent (thus the higher is U), the more additional consumption is needed to compensate the agent for the effort supply.¹⁴

Unemployment Insurance. We are now ready to characterize the value of UI for the planner.

$$\begin{aligned}
V^{UI}(U, h) &= \max_{c, U^f, U^s} -c + \beta \left[\pi(h) W(U^s, h) + (1 - \pi(h)) V(U^f, h) \right] \\
&\text{s.t.} \\
U &= \log(c) - e + \beta \left[\pi(h) U^s + (1 - \pi(h)) U^f \right], & \text{(PK)} \\
U &\geq \log(c) + \beta U^f, & \text{(IC-S)} \\
U^s &\geq U^f, & \text{(IC-R)} \\
U^s, U^f &\geq U. & \text{(NS)}
\end{aligned}$$

During UI , with probability $\pi(h)$ the agent finds a job in the market sector and W is the value associated to this outcome, evaluated at promised utility U^s . With probability $1 - \pi(h)$, the agent remains unemployed. The first constraint is again the promise-keeping constraint, while the second constraint, denoted as IC-S, is the incentive constraint inducing the agent to actively search for a job. The constraint denoted as IC-R represents the incentive constraint related to the retention action r . Recall that the planner observes neither the worker's search effort a , nor the retention action r during the job interview. In order to induce $r = 1$, the contract must guarantee an increase in utility upon success of the job interview.

Rearranging IC-S and using the PK constraint, we obtain:

$$U^s - U^f \geq \frac{e}{\beta\pi(h)} > 0, \quad (4)$$

which shows that the IC-S constraint implies that IC-R is satisfied as well. Finally, in the last row we have —as in all policies— the NS constraint guaranteeing the utility of the agent never decreases.

In Appendix A we show that the optimal solution delivers $U^f = U$ and $U^s = U + \frac{e}{\beta\pi(h)}$.

¹⁴This negative wealth effect is present whenever preferences over effort and consumption imply that leisure is a normal good. The limit case of this class is when preferences are $\log(c - e)$. Then, the effort compensation cost would be a fixed number, in consumption units, independent of the level of U .

Using this last equality into the PK constraint, and substituting the PK constraint into the IC-S constraint, we arrive at the consumption payments during UI

$$c^{UI} = \exp((1 - \beta) U).$$

Since $U^f = U$, if UI was optimal at (U, h) , it is optimal thereafter and $V = V^{UI}$. Using this result and substituting the expression for W into the definition of V^{UI} , we arrive at

$$V^{UI}(U, h) = \frac{1}{1 - \beta} \left[A^{UI}(h) - B^{UI}(h) \exp((1 - \beta) U) \right], \quad (5)$$

where the two coefficients are given by:

$$\begin{aligned} A^{UI}(h) &= \frac{\beta\pi(h)}{1 - \beta + \beta\pi(h)} \cdot \omega(h) \\ B^{UI}(h) &= \frac{1}{1 - \beta + \beta\pi(h)} \left[1 - \beta + \beta\pi(h) \exp\left(\frac{1 - \beta + \beta\pi(h)}{\beta\pi(h)} e\right) \right]. \end{aligned}$$

The coefficient A^{UI} reflects the expected gains from production that occur upon finding a job: in each period, with probability $\pi(h)$, the job-seeker finds employment in the market sector and produces $\omega(h)$ forever after. The coefficient B^{UI} —the cost of delivering utility during UI relative to SA —can be decomposed into two multiplicative terms:

$$B^{UI}(h) = \exp(e) \cdot \frac{\frac{1 - \beta}{1 - \beta + \beta\pi(h)} + \frac{\beta\pi(h)}{1 - \beta + \beta\pi(h)} \exp\left(\frac{1 - \beta + \beta\pi(h)}{\beta\pi(h)} e\right)}{\exp(e)}.$$

The first term is, once again, an effort compensation component: an agent in UI searches with high effort when unemployed and also exerts high work effort if job search leads to market-sector employment. The second term is induced by the incentive compatibility constraint IC-S, and it can be shown to be larger than 1 by Jensen's inequality.

The IC-S constraint entails a cost for the planner because, as shown in (4), it requires a minimum wedge between continuation utilities upon success U^s and upon failure U^f , and this wedge in turn requires different consumption payments in the two states. Risk-averse agents dislike consumption lotteries; thus implementing this lottery means that the planner must pay

a higher average overall level of consumption to deliver the same level of promised utility U compared to a situation where the constraint is not binding, such as SA or employment. We call this cost the *incentive cost*.

The incentive cost is decreasing in h . The reason is that, as seen in equation (4), the promised utility wedge between states ($U^s - U^f$) —and hence the incentives to search— must be large to induce an agent with a low skill level to search because, for this agent, the private returns to search are low. The fact that the incentive cost enters multiplicatively with respect to U means that this additional cost induced by the presence of the incentive compatibility constraint is higher the higher is U . This property can be related to the fact that the inverse function of the agent's utility $u^{-1}(\cdot) = \exp(\cdot)$ has a convex first derivative (e.g., [Pavoni and Violante, 2007](#)).

Finally, precisely because of the incentive constraint, it is easy to show that consumption increases when an unemployed worker of type (U, h) in UI finds a market-sector job. This jump in consumption is given by

$$\log \left[\frac{c^W(U^s(U, h), h)}{c^{UI}(U, h)} \right] = \log \left[\frac{\exp((1 - \beta)U^s + e)}{\exp((1 - \beta)U)} \right] = \frac{1 - \beta + \beta\pi(h)}{\beta\pi(h)}e > e > 0. \quad (6)$$

We now introduce the policy instruments generated by the use of the make-work production technology and by the assisted search technology.

Mandatory Work. The value of MW for the planner is given by

$$\begin{aligned} V^{MW}(U, h) &= \max_{c, U^f} \underline{\omega} - c + \beta V(U^f, h) \\ &\quad s.t. \\ U &= \log(c) - e + \beta U^f && \text{(PK)} \\ U^f &\geq U. && \text{(NS)} \end{aligned}$$

It is easy to show that, whenever MW is the optimal policy at (U, h) , consumption smoothing implies $U^f = U$, making this policy an absorbing state, like SA , with $c^{MW}(U, h) = \exp((1 - \beta)U + e)$. The value function can therefore be expressed as

$$V^{MW}(U, h) = \frac{1}{1 - \beta} \left[A^{MW}(h) - B^{MW}(h) \exp((1 - \beta)U) \right], \quad (7)$$

where

$$\begin{aligned} A^{MW}(h) &= \underline{\omega} \\ B^{MW}(h) &= \exp(e). \end{aligned}$$

Every period MW has a return $\underline{\omega}$ in terms of net output, which explains the A^{MW} coefficient. The coefficient B^{MW} indicates another important feature of MW , relative to SA . Delivering the promised utility U is more costly in consumption terms because the principal has to compensate the agent for the work effort, for the same reasons described in the market-employment problem. Note that since neither A^{MW} nor B^{MW} depend on h , the value of MW is only a function of U , and in what follows we denote it as $V^{MW}(U)$.

Job Search Assistance (JA). The value of JA for the planner is

$$\begin{aligned} V^{JA}(U, h) &= \max_{c, U^s, U^f} -c - \kappa + \beta \left[\pi(h) W(U^s, h) + (1 - \pi(h)) V(U^f, h) \right], \\ &\quad s.t. \\ U &= \log(c) + \beta \left[\pi(h) U^s + (1 - \pi(h)) U^f \right] && \text{(PK)} \\ U^s &\geq U^f && \text{(IC-R)} \\ U^s, U^f &\geq U. && \text{(NS)} \end{aligned}$$

Since search effort is set at zero ($a = 0$) in JA , we do not have the IC-S constraint in the problem. However, the agent must be induced to the high retention action upon meeting the potential employer, and this is the role played by the IC-R constraint. In Appendix A, we show that consumption smoothing across states implies that both the IC-R and NS constraints are binding at the optimum ($U^s = U^f = U$). Thus, if JA is optimal at (U, h) it remains optimal and $V = V^{JA}$.

The reason why the IC-R constraint is binding is that JA is a policy without agent's effort whereas a market-sector job requires effort. As a result, full consumption insurance across employment states necessarily implies a lower utility when employed. Since the agent can avoid employment by taking the low retention action $r = 0$, and this action is not observed by the planner, the IC-R constraint must bind to induce the action $r = 1$. In turn, the fact that the IC-R constraint is binding, means that consumption during employment must be large enough

to offset the additional disutility of work. Specifically, consumption payments under JA are equal to

$$c^{JA}(U, h) = \exp((1 - \beta)U),$$

and the consumption gap across employment states—the equivalent of equation (6) in UI —is

$$\log \left[\frac{c^W(U^s(U, h), h)}{c^{JA}(U, h)} \right] = \log \left[\frac{c^W(U, h)}{c^{JA}(U, h)} \right] = e > 0,$$

which reflects precisely the fact that employment requires e units of effort from the agent, while assisted search does not.

Once again, the value function for this policy can be written as:

$$V^{JA}(U, h) = \frac{1}{1 - \beta} \left[A^{JA}(h) - B^{JA}(h) \exp((1 - \beta)U) \right], \quad (8)$$

where

$$A^{JA}(h) = \frac{1}{1 - \beta + \beta\pi(h)} \cdot [\beta\pi(h)\omega(h) - (1 - \beta)\kappa]$$

$$B^{JA}(h) = \exp\left(\frac{\beta\pi(h)}{1 - \beta + \beta\pi(h)}e\right) \cdot \frac{\frac{1 - \beta}{1 - \beta + \beta\pi(h)} + \frac{\beta\pi(h)}{1 - \beta + \beta\pi(h)}\exp(e)}{\exp\left(\frac{\beta\pi(h)}{1 - \beta + \beta\pi(h)}e\right)} < \exp(e).$$

The coefficient A^{JA} is structured exactly as A^{UI} , since the job-finding rates and the expected returns are the same in both policies, with the addition of the cost κ that the planner incurs to use the search-assistance technology.

The B^{JA} coefficient can again be divided into two multiplicative components. The first component is the effort compensation cost that is paid by the planner only during employment, upon finding a job. Thus this component is smaller than its UI counterpart. As discussed above, JA has the incentive constraint IC-R stating that, in order to induce a positive retention action, the planner must generate some consumption dispersion across employment states. This incentive cost is captured by the second term in B^{JA} which is higher than one because $e > 0$. Finally, it is easy to see that, by Jensen's inequality, $B^{JA} < \exp(e)$, and thus the cost of delivering utility for the planner is smaller in JA than in MW and UI .

Transitional Work (TW). The *TW* policy consists of the joint use of search assistance and make-work technologies:

$$\begin{aligned}
V^{TW}(U, h) &= \max_{c, U^s, U^f} -c + (\underline{\omega} - \kappa) + \beta \left[\pi(h) W(U^s, h) + (1 - \pi(h)) V(U^f, h) \right] \\
&\quad s.t. \\
U &= \log(c) - e + \beta \left[\pi(h) U^s + (1 - \pi(h)) U^f \right] && \text{(PK)} \\
U^s &\geq U^f && \text{(IC-R)} \\
U^s, U^f &\geq U. && \text{(NS)}
\end{aligned}$$

Again, since both the IC-R and the NS constraints hold with equality, we have $U^s = U^f = U$ and, as a result, $V = V^{TW}$.¹⁵ Consumption is therefore:

$$c^{TW}(U, h) = \exp((1 - \beta)U + e) = c^W(U, h)$$

where the last equality implies that no upward jump in consumption is necessary when an agent in *TW* finds a job.

The value function associated to *TW* can be then written as:

$$V^{TW}(U, h) = \frac{1}{1 - \beta} \left[A^{TW}(h) - B^{TW}(h) \exp((1 - \beta)U) \right] \quad (9)$$

where

$$\begin{aligned}
A^{TW}(h) &= \frac{1}{1 - \beta + \beta\pi(h)} \cdot [\beta\pi(h)\omega(h) + (1 - \beta)(\underline{\omega} - \kappa)] \\
B^{TW}(h) &= \exp(e).
\end{aligned}$$

The first term of the A^{TW} coefficient is the same first term appearing in the constants of *UI* and *JA*: the expected return of job search. The second term captures the output from the make-work job net of the costs of operating the search assistance and secondary production technologies. The coefficient B^{TW} is the same as in *MW* and is associated to the effort compensation cost on

¹⁵In Appendix A we show that for combinations of (U, h) such that *TW* is optimally chosen, even though the IC-R constraints holds with equality, the multiplier associated to the constraint is zero.

	$A^i(h)$: return net of administrative cost	$B^i(h)$: cost of promising U in c terms
SA :	0	1
UI :	$\frac{\beta\pi(h)}{1-\beta+\beta\pi(h)}\omega(h)$	$\exp(e) \times \text{cost of the IC-S}$
MW :	$\underline{\omega}$	$\exp(e)$
JA :	$\frac{\beta\pi(h)}{1-\beta+\beta\pi(h)}\omega(h) + \frac{1-\beta}{1-\beta+\beta\pi(h)}(0 - \kappa)$	$\exp\left(\frac{\beta\pi(h)}{1-\beta+\beta\pi(h)}e\right) \times \text{cost of the IC-R}$
TW :	$\frac{\beta\pi(h)}{1-\beta+\beta\pi(h)}\omega(h) + \frac{1-\beta}{1-\beta+\beta\pi(h)}(\underline{\omega} - \kappa)$	$\exp(e)$

Table 3: Net returns and cost of delivering promised utility in each of the five policies. The cost is expressed in terms of the baseline cost during SA . Recall that $B^{JA} < \exp(e)$ and $B^{UI} > \exp(e)$.

the make-work job, during unemployment, and on the market-sector job upon finding employment.

Constant U . Collecting the results derived for each policy, we conclude that promised utility U is constant, and equal to its initial condition U_0 , for the entire duration of the optimal program. The reasons for this result differ somewhat among policies. MW and SA are absorbing policies without any incentive constraint. In TW , the IC-R constraint is present, but not binding. In these policies, perfect consumption smoothing is optimal, which implies constant consumption and constant U over time. In JA and UI , the incentive constraints are binding, but because of the NS constraint, the utility gap needed to satisfy the IC-R and IC-S constraints is achieved by setting $U^s \geq U^f = U$ (with the strict inequality holding in UI). As a result, U never changes during the program—it only rises when a worker finds a market-sector job from UI .

3.1 Comparison across policies

Table 3 summarizes the coefficients (A^i, B^i) of the value functions associated to the five policy instruments. These closed-form solutions are useful to rank policies with respect to their expected net returns and with respect to their cost of delivering promised utility relative to the baseline cost in SA . In the rest of the paper, we make the assumption that $\kappa > \underline{\omega} > 0$. This assumption guarantees that none of the policies are strictly dominated for all (U, h) and keep

the analysis as general as possible.¹⁶

A number of useful rankings across policies can therefore be established with respect to the net returns, for given U . The return in UI is larger than in JA because, even though the returns to search are the same in both policies, JA requires bearing the cost κ to operate the search assistance technology. The return in TW is also higher than the return in JA because the net return from the secondary production technology $\underline{\omega}$ is positive. SA has zero net returns, and thus, both MW and UI have higher returns than SA . However, one cannot rank the policies using search assistance with SA in terms of net returns because, for κ large enough and/or h small enough, the net return of JA and TW may be negative. The most interesting case is the one where the net returns of JA are positive at least for some h and, therefore, JA dominates SA in some range. Under this additional assumption, we have:

$$A^{UI}(h) > A^{TW}(h) > A^{JA}(h) > A^{SA} = 0 \text{ or } A^{UI}(h) > A^{MW} = \underline{\omega} > A^{JA}(h) > A^{SA} = 0.$$

Turning to the costs of delivering utility B^i , the model features an unambiguous ranking across all policies. SA is the policy where it is cheapest to deliver promised utility: its marginal cost is one. JA is next, since we showed that $B^{JA} < \exp(e)$, where $\exp(e)$ is the effort compensation cost present in MW and TW . Finally, UI is the most expensive policy for the planner because it features both the effort compensation cost and the incentive cost associated to satisfying the IC-S constraint. To sum up, we have:

$$B^{UI}(h) > B^{TW} = B^{MW} = \exp(e) > B^{JA}(h) > B^{SA} = 1,$$

These rankings of net returns and costs of delivering promised utility shape the trade-off among policies at every point in the (U, h) state space. Faced with a worker of type (U, h) , the planner solves

$$i(U, h) = \arg \max \left\{ V^{UI}(U, h), V^{SA}(U), V^{JA}(U, h), V^{MW}(U), V^{TW}(U, h) \right\}, \quad (10)$$

¹⁶Table 3 shows that if $\underline{\omega} \geq \kappa$, then TW dominates UI for all (U, h) , because returns are higher and costs are lower in TW . Similarly, if $\underline{\omega} \leq 0$, then TW is dominated by JA , and MW is dominated by SA over the entire state space.

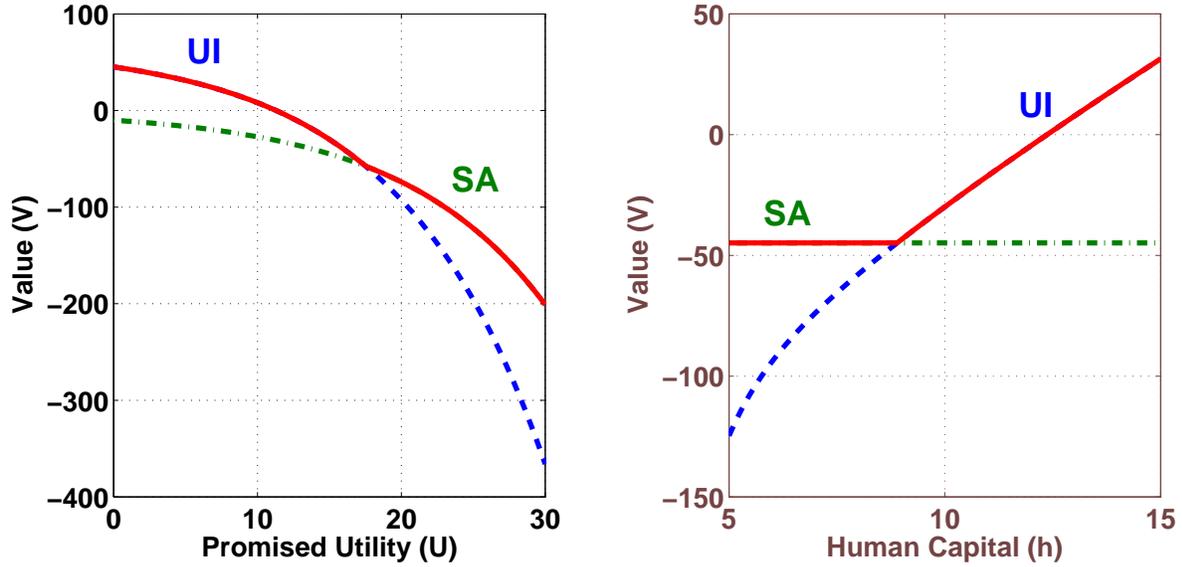


Figure 2: Comparison between the values of SA and UI . Left-panel: projection keeping h constant. Right-panel: projection keeping U constant. Solid line: upper envelope.

which means identifying, at every pair (U, h) , what policy dominates all the others.

We offer an example of this choice where the only two instruments available are UI and SA . The left panel in Figure 2 plots V^{UI} and V^{SA} as a function of U for given h .¹⁷ Both value functions are decreasing and concave. The value of SA is flatter with respect to U because of the absence of effort compensation and incentive costs in SA . Therefore, UI dominates only for small enough U . The right panel in Figure 2 plots V^{UI} and V^{SA} as a function of h for given U . The value function in SA is independent of h . The value of UI is, instead, increasing in h , but for low h this value can be very small because of the incentive cost that is larger for low h . Thus, UI dominates only for high enough h .

The solution to problem (10), projected onto the (U, h) state space, gives rise to the optimal welfare-to-work program, which we characterize next.

4 Policy space in the optimal welfare-to-work program

Figure 3 illustrates in which areas of the state space the different policy instruments arise as optimal. To fix ideas, the unit in the scale for h on the x axis corresponds to \$100 per month of

¹⁷As previously explained in footnote 13, we allow for randomizations across policies. In Appendix A, we show how this figure changes in the case with randomizations.

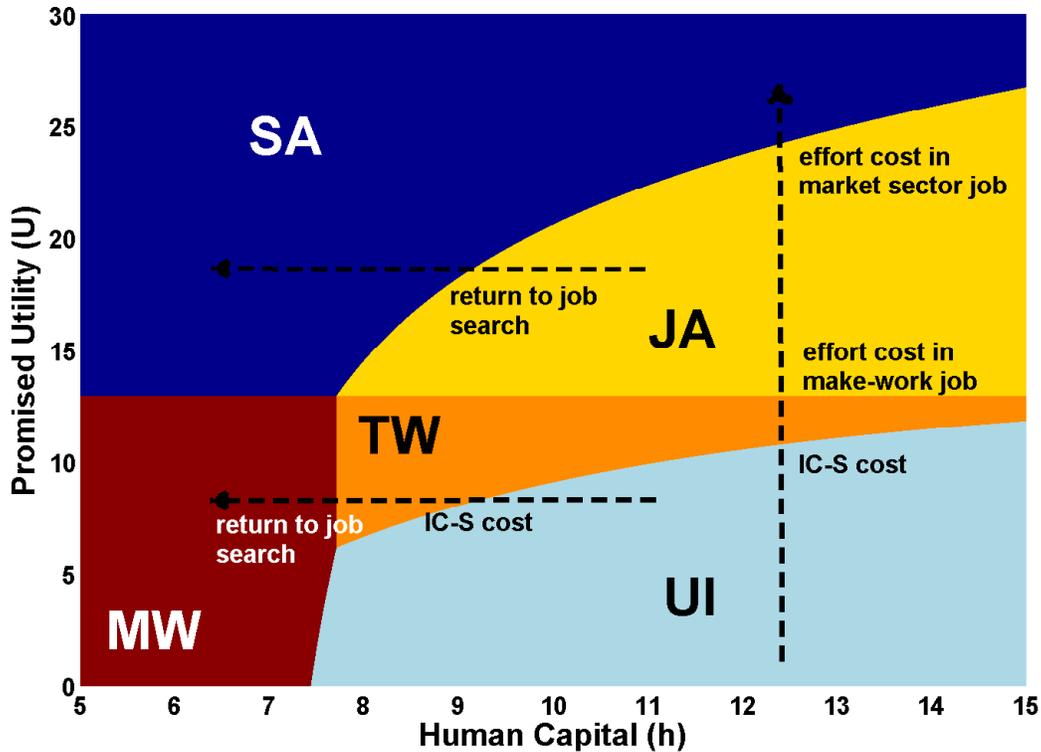


Figure 3: The horizontal axis variable h represents the participant's skill level (measured in hundreds of dollars a month), while the vertical axis variable U represents the level of generosity of the program. UI: Unemployment Insurance; JA: Job-search Assistance; TW: Transitional Work; MW: Mandatory Work; SA: Social Assistance.

earnings in a market-sector job, and 10 (20) units of U on the y axis corresponds to the promise of paying \$270 (\$740) of welfare benefits per month in SA during the program.¹⁸ Towards the right-hand of the figure we have skilled workers; towards the left the unskilled. Policies at the top of the figure are those adopted by generous governments, while the bottom of the figure features policies adopted by more parsimonious governments. Recall that both state variables, U and h , are constant along the program. This means that there is no transition between policies during the program. For a given level of program generosity U , and a given level of worker's skills h , the optimal program determines the unique policy instrument to be used until, possibly, the agent finds a market sector job.

To understand the relative position of the policies in the state space, Figure 3 illustrates the

¹⁸The parameter values used in this figure are: $\beta = 0.90$, $e = 0.67$, $\underline{\omega} = 3.5$, $\kappa = 5$. These last two parameters correspond to \$350 and \$500 per month, respectively. Moreover, the hazard function is $\pi(h) = 0.5 (h/h_{\max})^2$, and the wage function is $w(h) = h$.

forces at work for different levels of U and h .

Moving vertically (along U). Fix a large value of h towards the right side of the state space. For small enough values of U , UI is optimal. As U increases, the cost of satisfying the IC-S constraint becomes too large and the optimal program starts using the search assistance technology because its returns are equal to those in private search but its costs are increasing less sharply in U . At this point, promised utility is still low enough that it is worthwhile for the planner to require the agent to produce on the secondary technology and compensate her for the work effort, thus TW is used. For larger values of U , the effort compensation cost on the make-work job becomes too high, and the planner stops using the secondary production technology, and lets the agent use only the search-assistance technology that requires no search effort: JA becomes optimal. Finally, promised utility can be so high that it is even too costly to compensate the agent for her work effort upon finding a job in the market sector, given the output $\omega(h)$ produced at that given level of h . Then the optimal policy is SA , an absorbing policy of pure income support.

Moving horizontally (along h). Fix a high value of U such that JA is optimal. As worker's productivity h falls, the return to job search becomes so small that the planner does not find it optimal to pay the cost associated to assisted search and starts using SA . Now consider a lower value of U such that UI is optimal. As h falls, the incentive costs related to the IC-S constraint rise, and the planner switches to use assisted search. However, the level of promised utility is low enough that it keeps requiring the agent to exert high effort, no longer to search but to produce on the make-work job while the planner searches on her behalf. For low enough values of h , the return to job search becomes too low to justify the use of assisted search and the planner switches to MW , an absorbing policy where the agent is required to hold a make-work job in exchange for her welfare check.

Policy indifference curves. The boundaries between two policies are the policy-indifference curves obtained by solving for the function $U_{i,i'}^*(h)$ that equates the values of policy i and policy i' . For example, Figure 3 shows that the policy indifference curve between MW and SA is horizontal—because neither the costs nor the returns of these two policies depend on h —and equal to the policy indifference curve between TW and JA . Using the values of these policies,

it is easy to show that

$$U_{SA,MW}^* = U_{TW,JA}^* = \frac{1}{1-\beta} \log \left(\frac{\omega}{\exp(e) - 1} \right).$$

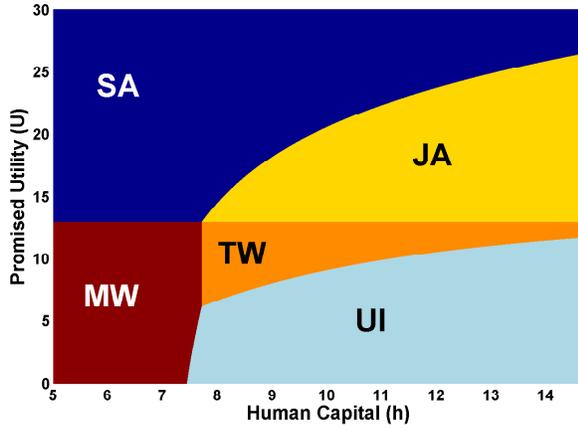
The policy indifference curve between *MW* and *TW* is vertical because the costs of delivering U are the same in both policies. The boundary between *JA* and *SA* is positively sloped because a rise in h increases the returns to search in *JA* and has no impact on the value of *SA*, but a rise in U increases the costs of *JA* faster than in *SA*. The boundary between *UI* and *MW* is positively sloped because a larger h increases the returns to search, reduces the incentive costs in *UI*, and has no impact on the value of *MW*, but a higher U increases the costs of delivering promised utility in *UI* faster than in *MW*. Finally, the positive slope between *TW* and *UI* is explained by the fact that, as U increases, the costs of *UI* rise faster —recall that $B^{UI}(h) > B^{TW}(h)$ — and, as h increases, the returns to search increase equally in both policies, but satisfying the IC-S constraint in *UI* becomes cheaper.

4.1 Comparative statics

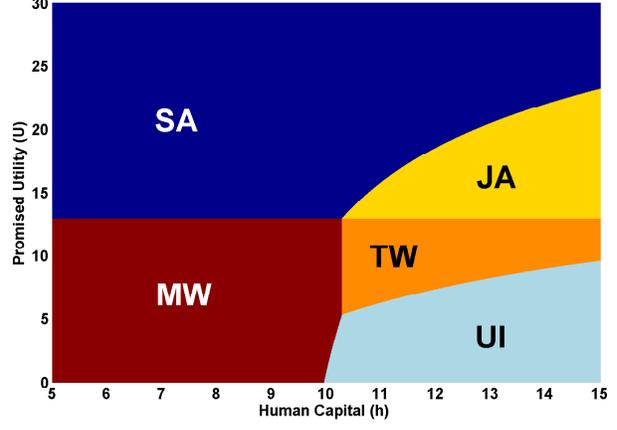
Figure 4 illustrates how the policy space changes with labor market conditions and with the effort level e .

Labor market conditions. In our model, labor market conditions are summarized by the pair of functions $\{\pi(h), w(h)\}$, the job finding rate and the wage. We think of tight vis-a-vis slack labor markets as situations where the two functions are shifted up and down, respectively: in a tight labor market a worker finds a job faster and earns more, for any given h . To understand the implications of labor market conditions for the design of welfare programs, the top panels of Figure 4 compare a tight labor market to a slack labor market.

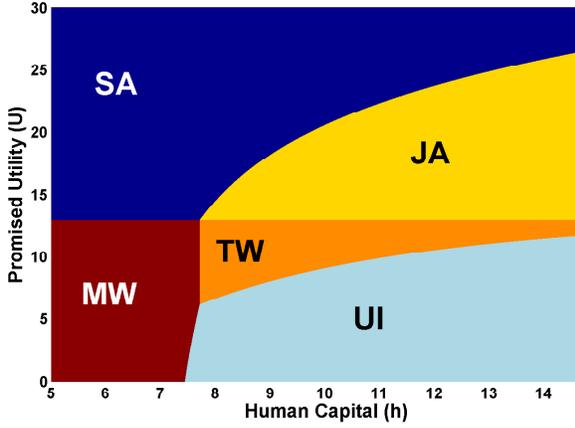
Because of the lower return to job search, the optimal program makes heavier use of *SA* and *MW* in a slack labor market. At the same time, whenever private search is implemented in the program, incentive compatibility requires a larger promised utility gap to induce the worker to exert high effort. Recall that the wage tax (subsidy, if negative) rate for an agent who finds a



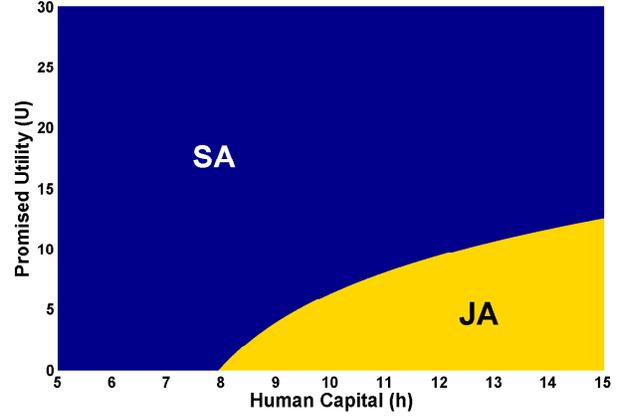
(a) Tight Labor Market



(b) Slack Labor Market



(c) Low Effort Cost



(d) High Effort Cost

Figure 4: Top panels: comparative statics with respect to labor market conditions $\pi(h), \omega(h)$. Bottom panels: comparative statics with respect to the value of the (search and work) effort cost. UI: Unemployment Insurance; JA: Job-search Assistance; TW: Transitional Work; MW: Mandatory Work; SA: Social Assistance.

market-sector job from *UI* is

$$\tau^{UI}(U, h) = 1 - \frac{c^W(U^s(U, h), h)}{\omega(h)} = 1 - \frac{1}{\omega(h)} \cdot \exp\left((1 - \beta)U + \frac{1 - \beta + \beta\pi(h)}{\beta\pi(h)}e\right).$$

The tax (subsidy) is therefore increasing (decreasing) in π and ω . Thus, the optimal design of welfare programs in slack labor markets calls for the use of lower wage taxes or higher wage subsidies for *UI* recipients.

Even though we have not solved the model with shocks to labor market conditions, one should expect that in such a version of the model, a change in the state of the labor market

Consumption during the program	Consumption during employment
$c^{SA} = \exp((1 - \beta) U)$	
$c^{UI} = \exp((1 - \beta) U)$	$c^W = \exp\left((1 - \beta) U + \frac{1 - \beta + \beta\pi(h)}{\beta\pi(h)} e\right)$
$c^{MW} = \exp((1 - \beta) U + e)$	
$c^{JA} = \exp((1 - \beta) U)$	$c^W = \exp((1 - \beta) U + e)$
$c^{TW} = \exp((1 - \beta) U + e)$	$c^W = \exp((1 - \beta) U + e)$

Table 4: Consumption in each policy during the program (left column) and upon employment for those policies that contemplate job search (right column).

induces policy transitions during the program. Consider, for example, an agent who is at a point (U, h) of the state space that belongs to UI but is near the border between UI and MW . If a recession hits the economy, and the labor market turns from tight to slack, the planner may find it optimal to switch policy and enroll that same worker in MW activities instead of asking that she continues to search for a job.

Effort cost. The effort cost e might differ a great deal across demographic groups. For example, for single parents of young children or disabled individuals, job search and work can be a lot more costly than for other groups in the population. As the bottom panels of Figure 4 shows, the optimal welfare program should vary across demographic group with different effort cost. When dealing with high-cost individuals, the program should more heavily use policies that do not require effort: JA for low levels of U and high levels of h , where assisted job search still has positive returns, and SA where the returns from job search are too small.

4.2 Robustness to hidden saving

So far we have solved the principal-agent problem under the assumption that savings are observable to the planner. We now relax this assumption and extend our analysis to the case where the agent's savings, with rate of return $\beta^{-1} - 1$, are also subject to a private information problem. We maintain the assumptions, stated earlier, that the unemployed starts her spell with no wealth and cannot borrow.

The first question to ask is: does the agent ever have an incentive to save along the program? As explained, since promised utility is constant, and there is no policy transition within the program, consumption is also constant during unemployment. The agent's consumption changes

only if she finds a job. Table 4 summarizes consumption payments during each policy, and upon finding a market-sector job from those policies that use search (UI, JA, TW). This table shows that, when switching to market employment from UI, JA , or TW , consumption never drops. As a result, the agent never has an incentive to save along the program —she would like to borrow to smooth consumption across employment states, but the borrowing constraint prevents her from doing so.

Stated differently, the solution to the relaxed problem, where the observable saving constraint is not imposed, is also the optimal contract for the restricted problem since in the relaxed problem the constraint remains slack. This property follows from the no-stick constraint. We do recognize that the no-stick constraint is restrictive, and reduces the potential value of the policies with binding incentive constraints such as UI . At the same time, our result suggests that ruling out from the contract sanctions in the form of benefit reductions has the advantage of eliminating the hidden-saving problem.

In more general environments, where the dynamic provision of incentives induces a decline in promised utility and in consumption over time, the agent may have an incentive to save.¹⁹ However, in some cases, even without any restrictions on punishments, the optimal contract entails constant benefits. For example, within a pure UI framework *where agents have access to a riskless asset*, Shimer and Werning (2008) show that when workers have exponential utility (CARA), a simple contract such as the one we propose here, namely constant benefits during unemployment and a constant tax during employment, is optimal. They also show that, with CRRA utility, the optimal contract involves nearly constant benefits and more elaborate benefit structures offer only small welfare gains.

4.3 Human capital depreciation: a first look

Up to this point, we have thought of h as a fixed individual type. An alternative view is that h is a stock of human capital that depreciates along the non-employment spell. Human capital

¹⁹When the agent can privately save, the joint shirk & save deviation constitutes the relevant deviation in many dynamic moral hazard environments (e.g., Kocherlakota, 2004). Note that in our model, when the agent shirks, s/he remains unemployed with probability one. Thus, the relevant Euler equation for the shirking agent is: $u'(c_t) \geq u'(c_{t+1})$. It is hence clear that, whenever the program recommends decreasing consumption during unemployment, the shirking agent would want to save.

depreciation has two implications. Since the wage function $\omega(\cdot)$ depends on h , this generalization allows for a decline in earning power during non-employment, an idea supported by the empirical literature on earnings losses upon displacement —starting from [Mincer and Ofek \(1982\)](#)— which shows that these losses increase with the duration of the unemployment spell. Because $\pi(\cdot)$ depends on h , skill depreciation also means that the unemployment hazard displays negative duration dependence.

The planner’s problems under the various policies are exactly as described in [Section 3](#), with the crucial difference that, in each policy, the arguments of the continuation values $V(\cdot)$ are (U^f, h') instead of (U^f, h) , with $h' \leq h$. [Pavoni, Setty, and Violante \(2014\)](#) offer a characterization of the optimal welfare program when h decays along the unemployment spell. The rest of this section borrows heavily from that paper. There, we show that in spite of the fact that skill depreciation affects the values of some policies, the state space does not change much relative to the case with fixed skill levels. Thus, to describe the optimal WTW program in this case, we can use again the policy space represented in [Figure 3](#).

When h falls during unemployment, [Figure 3](#) becomes a *phase diagram* where workers on the welfare program transit from right to left —as their skills decay— at a constant level of continuation utility, the initial value U promised by the program.²⁰ The comparative static analysis on h discussed in [Section 4](#) (horizontal movements from right to left) suggests that two types of welfare programs emerge as optimal, depending on the level of generosity of the program U . A generous (or deep pocketed) principal would implement an optimal program featuring the ‘effort-free’ policy sequence $JA \rightarrow SA$. A more parsimonious (or more budget constrained) principal would, instead, implement an ‘effort-intensive’ program that follows the sequence $UI \rightarrow TW \rightarrow MW$, and would skip TW for especially low levels of U . The set of policy transitions featured by optimal programs is thus very limited. In particular, there would never be a transition originating from SA or from MW , and never a transition into UI .

5 An exploratory analysis of the data on US states

We motivated the paper by showing how heterogeneous the design of welfare programs is across US states —recall [Table 1](#). In the normative framework we developed, the key deter-

²⁰Although with skill depreciation it is not immediate to exclude that U^f is optimally higher than U in some states, here we abstract from this possibility and impose $U^f = U$ throughout the program.

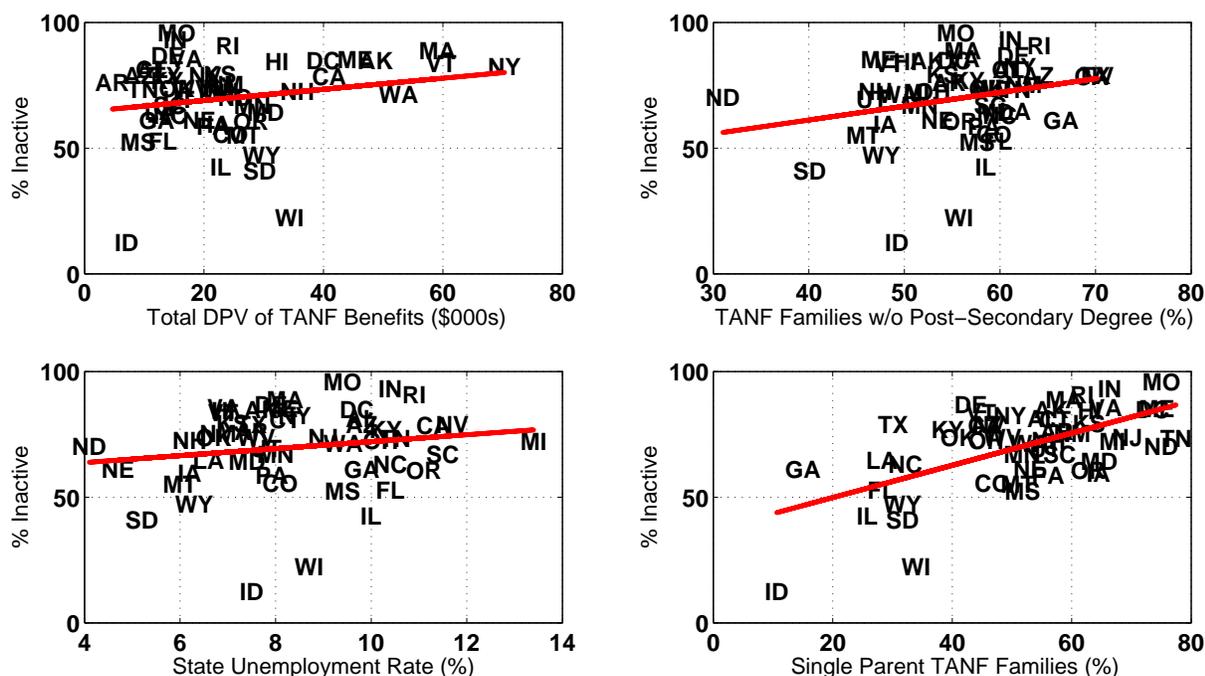


Figure 5: Percentage of inactive welfare recipients (not searching, working, or training) plotted against indicators of program generosity (top-left), participants skill level (top-right), local labor market tightness (bottom-left), and effort cost (bottom-right). Each point in the scatterplot is a U.S. state. The line on each panel is the OLS regression line.

minimants of the design of a welfare program that can, potentially, account for this heterogeneity are: the available budget per participant, the skill distribution of the participants, the local labor market conditions, and the distribution of effort costs in the population such as marital status, number of children, and disability status. In particular, the optimal design prescribes that states that pay higher TANF benefits, those whose labor force is less skilled, those with slacker labor markets, and those where single parents with young children are most prevalent should be the ones that tolerate more inactivity — Social Assistance in the model— among their welfare participants.

Figure 5 displays four simple scatterplots where state-level proxies for these four variables are plotted jointly with the fraction of TANF recipients in each state.²¹ As a proxy for the level of generosity, we computed the maximum lifetime discounted present value (DPV) of TANF

²¹Appendix B contains more details on the construction of the data used in this section.

Regressor	OLS coefficient (S.E.)
Constant	-24.74 (15.80)
DPV of TANF Benefits	0.21 (0.12)
% of Families w/o Secondary Degree	-0.88 (1.03)
State Unemployment Rate (%)	1.07 (0.28)
% of Single-Parent TANF families	0.71 (0.11)

Table 5: Multivariate regression. Dependent variable: % of inactive TANF participants

benefits calculated as monthly benefits for a family of three (single parent with two children) with no income times the lifetime limit in months, discounted at an annual rate of 5 percent. The top-left panel shows that the dispersion of monthly benefits documented in Figure 1 gets further amplified when time limits are taken into account, as time limits and monthly benefits are positively correlated (the correlation coefficient is 0.34). As a measure of the skill level, we take the fraction of TANF households without a post-secondary degree. The top-right panel shows a large variation in this variable across US states, ranging from 30 to 70 percent. To capture the local labor market tightness, we use the state unemployment rate in 2009. Once again, the dispersion across states is substantial, from the 4 percent of North Dakota to the 14 percent of Michigan. Finally, as an indicator of the distribution of effort costs, we take the fraction of single-parents on welfare which, as illustrated in the bottom-right panel, displays significant variation among US states: in some states, such as Georgia and Idaho, less than one-fifth of TANF households are single parent, whereas in others such as Missouri and Tennessee close to three-quarters of them are single parents.

Figure 5 shows that we find a positive correlation between the fraction of inactive TANF households and each of these four state-level indicators, as predicted by the theory. Table 5 shows that, when these four indicators are used jointly as explanatory variables in a cross-state linear regression model, all our indicators are statistically significant at least at the 10 percent level, with the exception of the skill distribution.

However, a large amount of cross-state variation remains unexplained: many states have similar levels of generosity, education composition, unemployment rate, and household composition, but very different fractions of welfare recipients who are not required to search, work, or train by the state. Given the roughness of our proxies, such large residual is far from surprising. At the same time, it could suggest that many states can save considerably by appropriately

redesigning the combination of instruments used in their welfare-to-work programs, without any utility loss for participants.

6 Conclusions

This paper studies the optimal design of ‘soft’ welfare-to-work programs, i.e., programs that do not utilize punishments (stick) but only rewards (carrot) in order to deliver incentives to the participants. We have modeled such restriction on the contract by imposing an additional constraint to the problem—the ‘no-stick’ constraint—stating that the continuation utility promised by the principal cannot decline along the program. The advantage of focusing the analysis on this class of contracts is twofold. First, analytical closed-form expressions can be obtained for all the value functions, and one can therefore describe very transparently the trade-offs among the various policy instruments in the planner’s menu of feasible options. Second, ‘soft’ welfare programs are immune to the hidden saving friction when the principal cannot observe the agent’s assets: consumption is weakly increasing along the program and, as a result, participants do not desire to save for the future.

A natural question is: how restrictive is this class of contracts, relative to the fully optimal contract where the welfare program contemplates the provision of dynamic incentives by reducing promised utility along the program? In [Pavoni, Setty, and Violante \(2014\)](#), we solve numerically for the optimal welfare program that comprises all the technologies (and the corresponding policy instruments) analyzed here, and conclude that the policy space is qualitatively very similar to the one emerging here. The reason is that, during policies where continuation utility optimally falls over time (Unemployment Insurance and Job-search Assistance), consumption smoothing dictates that, quantitatively, such decline should be small, and hence violations of the no-stick constraints are minor.

Going forward, a number of important questions remain unexplored and should be addressed in future work. First, are actual welfare-to-work programs observed across US states optimally designed? And, if not, how large are the budget savings of switching from the actual to the optimally designed program? In [Section 5](#) we have only scratched the surface of this question with a suggestive reduced-form empirical analysis.

The framework analyzed here does not allow the principal to use a training technology that rebuilds skills. In [Pavoni and Violante \(2005\)](#) we made an early attempt to model training, and a more recent example is [Spinnewijn \(2013\)](#). A lot remains to be done to integrate the various forms of training-based policies observed in the data (i.e., vocational training, basic schooling, and on-the-job training) within our framework.

Finally, here we have kept to the tradition of the optimal unemployment insurance literature and focused on unobservable effort—and moral hazard—as the source of private information. In the Mirrleesian tradition, the problem of the optimal design of welfare programs can also be approached from the perspective of adverse selection due to hidden types. In the context of our framework, one could introduce unobserved heterogeneity in the effort cost, or assume that individual human capital is unobservable. [Besley and Coate \(1992\)](#) takes this approach in a static setting and represents a useful starting point for a dynamic analysis.

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APPENDIX

This Appendix is organized as follows. Section A contains a detailed derivation of the closed forms for all the value functions, and explains the randomization used to convexify the problem. Section B describes the sources for the data used in Table 1 and Figure 5.

A Proofs of the closed-form value functions

For ease of exposition, the value functions derived in the text correspond to the upper envelope V when randomizations are not used. Here, we explicitly recognize that the planner may find it optimal to use lotteries across policies. We will therefore also need to characterize the ranges of utilities over which randomizations are used, and the exact slope of the value function in that range, which will be a constant since, as we will show, V is linear in the randomization region.

A.1 Non-randomization regions

We begin by showing that, whenever we are in non-randomization ranges, in all cases we obtain $U^f = U$, and the incentive constraints are binding. This implies that U remains constant from then on, and V will be strictly concave, possibly with the exclusion of period zero, depending on the initial conditions U_0 . The $t = 0$ case must be therefore studied by allowing for lotteries, and we do it in the next section.

For now, assume we are in point U where lotteries are not used. Let's consider each policy in turn.

Social Assistance:

$$V^{SA}(U, h) = \max_{c, U^f} -c + \beta V(U^f, h) \\ s.t.$$

$$U = \log(c) + \beta U^f, \quad (\text{PK})$$

$$U^f \geq U. \quad (\text{NS})$$

We guess and verify our closed form (1). In the relaxed problem where NS is ignored, we have

as necessary and sufficient condition:

$$V_U^{SA}(U, h) = V_U(U^f, h).$$

Moreover, if SA is the optimal policy choice, we have $V_U^{SA}(U, h) = V_U(U, h)$ and thus, by concavity, $U^f = U$ is an optimal solution. This solution satisfies NS, and therefore it is an optimal solution to the original problem as well.

Market-sector employment:

The proof is immediate; recall indeed that there is no transition to any other policy by assumption.

Unemployment Insurance:

$$\begin{aligned} V^{UI}(U, h) &= \max_{c, U^f, U^s} -c + \beta \left[\pi(h) W(U^s, h) + (1 - \pi(h)) V(U^f, h) \right] \\ &\text{s.t.} \\ U &= \log(c) - e + \beta \left[\pi(h) U^s + (1 - \pi(h)) U^f \right], & \text{(PK)} \\ U^s &\geq U^f + \frac{e}{\beta \pi(h)}, & \text{(IC-S)} \\ U^f &\geq U, & \text{(NS)} \end{aligned}$$

where we can neglect the redundant constraints IC-R and $U^s \geq U$. We aim at showing, by contradiction, that one cannot have $U^f > U$. Suppose $U^f > U$. Then NS is slack and the FOCs, together with the envelope condition, imply:

$$W_U(U^s, h) \leq V_U^{UI}(U, h) \leq V_U(U^f, h), \quad (\text{A1})$$

where the inequalities are strict whenever the incentive constraint IC-S binds with a positive multiplier. As a preliminary observation, note that since U is assumed to be optimal for UI we have $V_U^{UI}(U, h) = V_U(U, h)$. (i) Suppose first that the multiplier associated to the IC-S constraint is positive. Then the inequalities in (A1) are all strict. From the concavity of V it must be that $U^f < U$, hence a contradiction to the assumption. (ii) Assume now the multiplier associated to IC-S is not binding so that all conditions in (A1) are with equality (of course, the

only possibility of getting $U^f > U$ without violating the previous optimality conditions is that V is linear). We exclude the possibility that $U^f > U$ with a proof by contradiction. From the PK constraint, we get $-c = -\exp(U - \beta(1 - \pi(h))U^f + e - \beta\pi(h)U^s)$. The FOC w.r.t. U^s , assuming IC-S is slack, reads:

$$\exp(U - \beta(1 - \pi(h))U^f + e - \beta\pi(h)U^s) = W_U(U^s, h) = \exp((1 - \beta)U^s + e),$$

which implies $U - \beta(1 - \pi(h))U^f - \beta\pi(h)U^s = (1 - \beta)U^s$. However, this cannot be true since, as a consequence of our assumptions, $U - \beta(1 - \pi(h))U^f - \beta\pi(h)U^s < (1 - \beta(1 - \pi(h)))U^f - \beta\pi(h)U^s \leq (1 - \beta)U^s$. This contradicts the joint assumption IC-S slack and $U^f > U$. Finally, it is easy to see that whenever $U = U^f$, IC-S must be binding. If it were not the case, combining the FOC with PK would imply $U^s = U^f$, which violates IC-S. Therefore, IC-S must have a positive multiplier.

Mandatory Work:

$$V^{MW}(U, h) = \max_{c, U^f} \underline{\omega} - c + \beta V(U^f, h)$$

s.t.

$$U = \log(c) - e + \beta U^f, \quad (\text{PK})$$

$$U^f \geq U. \quad (\text{NS})$$

This proof follows closely the one outlined for SA. Solving the relaxed problem where NS is ignored, we have as necessary and sufficient condition:

$$V_U^{MW}(U, h) = V_U(U^f, h).$$

If MW is the optimal choice of policy, we have $V_U^{MW}(U, h) = V_U(U, h)$ and hence, by concavity, we have that $U^f = U$ is an optimal solution. This solves the problem with NS, and therefore it is an optimal solution to the original problem.

Job Search Assistance:

$$\begin{aligned}
 V^{JA}(U, h) &= \max_{c, U^s, U^f} -c - \kappa + \beta \left[\pi(h) W(U^s, h) + (1 - \pi(h)) V(U^f, h) \right], \\
 &\quad \text{s.t.} \\
 U &= \log(c) + \beta \left[\pi(h) U^s + (1 - \pi(h)) U^f \right], & \text{(PK)} \\
 U^s &\geq U^f, & \text{(IC-R)} \\
 U^f &\geq U, & \text{(NS)}
 \end{aligned}$$

where we can neglect the constraint $U^s \geq U$ since it is implied by IC-R together with NS.

Since the inequalities in (A1) do not depend on the exact nature of the incentive constraint, it is immediate to follow exactly the same line of proof as in the first part of *UI* to show the result $U^f = U$ for *JA* when the multiplier associated to IC-R is positive. Assume now the multiplier associated to IC-R is not binding. If we use PK, we obtain the following expression $-c = -\exp(U - \beta(1 - \pi(h))U^f - \beta\pi(h)U^s)$. By assumption both constraints are slack, so $U^s \geq U^f > U$. Recalling that the analogous of (A1) are with equality, the first order condition w.r.t. U^s reads:

$$\exp\{U - \beta(1 - \pi(h))U^f - \beta\pi(h)U^s\} = W_U(U^s, h) = \exp\{(1 - \beta)U^s + e\},$$

where we used the expression for W . Since $e > 0$ it must be that $U - \beta(1 - \pi(h))U^f - \beta\pi(h)U^s > (1 - \beta)U^s$. On the other hand, from the conditions $U^f > U$ and $U^s \geq U^f$, we get the contradiction: $U - \beta(1 - \pi(h))U^f - \beta\pi(h)U^s < (1 - \beta(1 - \pi(h)))U^f - \beta\pi(h)U^s \leq (1 - \beta)U^s$. So, in both cases we have a contradiction. It is easy to see from the proof that the only possibility is that the multiplier on IC-R is positive.

Transitional Work:

$$\begin{aligned} V^{TW}(U, h) &= \max_{c, U^f, U^s} -c + (\underline{\omega} - \kappa) + \beta \left[\pi(h) W(U^s, h) + (1 - \pi(h)) V(U^f, h) \right] \\ &\quad \text{s.t.} \\ U &= \log(c) - e + \beta \left[\pi(h) U^s + (1 - \pi(h)) U^f \right], & \text{(PK)} \\ U^s &\geq U^f, & \text{(IC-R)} \\ U^f &\geq U. & \text{(NS)} \end{aligned}$$

Assuming NS is slack, the necessary and sufficient conditions for optimality are:

$$W_U(U^s, h) \leq V_U^{TW}(U, h) \leq V_U(U^f, h), \quad (\text{A2})$$

with all inequalities strict whenever the multiplier associated to IC-R is positive. If IC-R has a positive multiplier, since by assumption $V_U^{TW}(U, h) = V_U(U, h)$ concavity implies that $U^f < U$ —hence a contradiction. Assume IC-R is slack and $U^f > U$. Under the same line of proof we followed for UI , we get that again this would lead to a contradiction. Hence it must be that $U^f = U$. In this case, setting $U^f = U^s$ is also optimal and the multiplier associated to IC-R is zero. This concludes the proof. **Q.E.D.**

A.2 The randomization problem

Without loss of generality, we assume that, when lotteries are used, in period zero the planner randomizes across policies, and then makes the appropriate payment based on the outcome of the lottery. It is easy to show that the planner does not benefit from randomizing across more than two policies.

Let U be an initial value of continuation utility such that it is optimal to randomize between policy i and j . To save on notation, we neglect the h variable. Then, the value for the planner at

U is given by:

$$V(U) = \max_{\theta \in [0,1], U^i, U^j} \theta \left[\frac{A^i}{1-\beta} - \frac{B^i}{1-\beta} \exp \left((1-\beta) U^i \right) \right] + (1-\theta) \left[\frac{A^j}{1-\beta} - \frac{B^j}{1-\beta} \exp \left((1-\beta) U^j \right) \right]$$

s.t.

$$U = \theta U^i + (1-\theta) U^j \quad (\phi)$$

Without loss of generality, we assume $A^i > A^j$ and $B^i > B^j$. We look for interior solutions for $\theta \in (0, 1)$. It is easy to see that the problem is concave. The necessary and sufficient FOCs are:

$$\begin{aligned} \theta & : & A^i - A^j - B^i \exp \left((1-\beta) U^i \right) + B^j \exp \left((1-\beta) U^j \right) &= (1-\beta) \phi \left(U^i - U^j \right) \\ U^i & : & B^i \exp \left((1-\beta) U^i \right) &= -\phi \\ U^j & : & B^j \exp \left((1-\beta) U^j \right) &= -\phi \\ \text{ENV.} & : & V'(U) &= \phi. \end{aligned}$$

Note that in the first FOC we assumed that θ is interior, but it is easy to account for corners. The optimality conditions imply that for an interior solution, we have:

- (i) Combining the FOC w.r.t. $U^i, U^j \Rightarrow (1-\beta)(U^j - U^i) = \ln \frac{B^i}{B^j} > 0$;
- (ii) From the FOC w.r.t. $\theta \Rightarrow -\phi = \frac{A^i - A^j}{(1-\beta)(U^j - U^i)} = \frac{A^i - A^j}{\ln \frac{B^i}{B^j}} > 0$ (Independent on U);
- (iii) Using each FOC, for $s = i, j$: $\exp \left((1-\beta) U^s \right) = \frac{1}{B^s} \frac{A^i - A^j}{\ln \frac{B^i}{B^j}} > 0$;

The function V is hence linear for a range of utilities $(1-\beta) U \in \left[\ln \left(\frac{1}{B^i} \frac{A^i - A^j}{\ln \frac{B^i}{B^j}} \right), \ln \left(\frac{1}{B^j} \frac{A^i - A^j}{\ln \frac{B^i}{B^j}} \right) \right]$, with randomization probability (expression obtainable from the ϕ constraint)

$$\theta^*(U) = \frac{U - U^j}{U^i - U^j} = \frac{\ln \left(\frac{1}{B^j} \frac{A^i - A^j}{\ln \frac{B^i}{B^j}} \right) - (1-\beta) U}{\ln \frac{B^i}{B^j}}. \quad (\text{A3})$$

To summarize our result, for a fixed h the function $V(\cdot, h)$ can be described as follows:

1. We have at most 4 intervals in $(1 - \beta)U$ defined by $\left[\ln \left(\frac{1}{B^i} \frac{A^i - A^j}{\ln \frac{B^i}{B^j}} \right), \ln \left(\frac{1}{B^j} \frac{A^i - A^j}{\ln \frac{B^i}{B^j}} \right) \right]$, and all the intervals defining the complementary set of them to get $(-\infty, +\infty)$.
2. Outside these intervals, if not overlapping with other intervals, V takes the shape described in the main text. In particular:

- for values $(1 - \beta)U \geq \ln \left(\frac{1}{B^j} \frac{A^i - A^j}{\ln \frac{B^i}{B^j}} \right)$ the function V equals:

$$V(U) = \frac{A^j}{1 - \beta} - \frac{B^j}{1 - \beta} \exp((1 - \beta)U),$$

- and for values $(1 - \beta)U \leq \ln \left(\frac{1}{B^i} \frac{A^i - A^j}{\ln \frac{B^i}{B^j}} \right)$ the expression becomes:

$$V(U) = \frac{A^i}{1 - \beta} - \frac{B^i}{1 - \beta} \exp((1 - \beta)U).$$

3. Inside the intervals, the value function is linear with slope $\phi = -\frac{A^i - A^j}{\ln \frac{B^i}{B^j}}$, and it takes the following expression (note that for $s = i, j$, we have $B^s \exp((1 - \beta)U^s) = \frac{A^i - A^j}{\ln \frac{B^i}{B^j}}$)

$$V(U) = \theta^*(U)V^i(U^i) + (1 - \theta^*(U))V^j(U^j) = \frac{\theta^*(U)A^i + (1 - \theta^*(U))A^j - \frac{A^i - A^j}{\ln \frac{B^i}{B^j}}}{1 - \beta}$$

or, using the expression (A3) for $\theta^*(U)$

$$V(U) = \frac{\gamma A^i + A^j(1 - \gamma)}{(1 - \beta)} - \frac{A^i - A^j}{\ln \frac{B^i}{B^j}} U, \quad \text{where } \gamma = \frac{\ln \left(\frac{1}{B^j} \frac{A^i - A^j}{\ln \frac{B^i}{B^j}} \right) - 1}{\ln \frac{B^i}{B^j}}.$$

4. If two or more intervals overlap, then some of the policies are dominated by the randomization and are not contemplated in the policy space.

What would remain to show is that V is not used in the randomization, unless it takes the

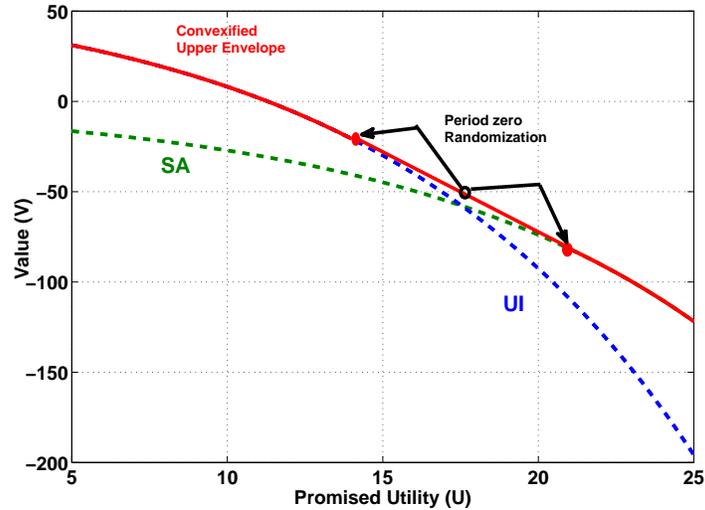


Figure A1: Convexified upper envelope V when the planner's choice is between UI and SA at time zero.

expressions in the main text. This is easy to show since V is linear in these ranges; hence not using it is weakly optimal. In other terms, the use of extreme values is sufficient for optimality, but it might not be necessary (compare [Abreu, Pearce, and Stacchetti, 1990](#)).

We conclude by illustrating, graphically, how lotteries change the shape of V . Figure A1 is the counterpart of the left-panel of Figure 2 in the main text, when the planner initially randomizes between UI and SA . Note the flat region of V , corresponding to the range of U for which randomizations are used. The outcome of the time-zero lottery forces the agent to start either on the extreme left of the flat region (hence in UI with a lower promised utility) or on the extreme right of the flat region (and hence in SA with a higher promised utility).

B Data used in Tables 1 and 5, and in Figure 5

The data on the fraction of inactive participants is computed from Table 6.A, "Average monthly number of work-eligible individuals with hours of participation in work activities, Fiscal Year 2009," produced by the Department of Health and Human Services, among the tables describing Work Participation Rates for the Fiscal Year 2009, and available from the U.S. Department of Health and Human Services at <http://www.acf.hhs.gov/programs/ofa/resource/wpr2009>.

The number of total participants is computed as total number of work-eligible individuals (WEI), i.e. adults receiving TANF assistance, minus those in unsubsidized private employment. Among those, the number of active participants is computed as total number of WEI with hours of participation, excluding those in unsubsidized private employment. The number of inactive participants is total participants minus active ones.

Individuals in work activities are taken to be those in subsidized public employment, work experience, and community service. Individuals in training are those in on-the-job training, vocational education, job-skill training, education-related to employment, and satisfactory school attendance. Individuals in job search activities are those in job search.

Data on the DPV of TANF benefits in Figure 5 are computed as maximum monthly benefits for a family of three without income times the maximum lifetime limit for each state, discounted at an annual rate of 5 percent. The federal government imposes a maximum 60 month lifetime limit on receipt of TANF benefits by adults. Therefore, after 60 months of receiving federally funded TANF benefits, either consecutively or nonconsecutively, a recipient is generally no longer eligible for federal cash assistance. Some states have adopted shorter lifetime limits (e.g., Arizona and Arkansas), while others have chosen to fund recipients after the 60 months with state funds (e.g., Maine and Massachusetts). In these few cases, we have arbitrarily set the time limit to twice the federal limit, but results are not sensitive to this assumption.

The source for data on the fraction of poor working families where no parent has some post-secondary education (year 2009) is <http://www.clasp.org/data>.

Data on the 2009 statewide unemployment rate are available through the Bureau of Labor Statistics through its Local Area Unemployment Statistics Program, <http://www.bls.gov/lau/>.

Data on the fraction of single-parent TANF families (year 2009) are computed from tabulations based on TANF caseload data by state available from the U.S. Department of Health and Human Services at <http://www.acf.hhs.gov/programs/ofa/resource/caseload-data-2009>.