Moral Hazard and Endogenous Monitoring

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Abstract
I study a principal-agent problem with monitoring where the principal chooses the signal's precision of the agent's action. I use the model to study how the principal's monitoring choice depends on each of two properties of the agent: his disutility from performing the task and his probability of succeeding in the task given positive effort.

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1 Introduction

In the canonical principal-agent model, a risk-neutral principal provides to a risk-averse agent a transfer that depends on the outcome. That noisy signal induces uncertainty in the agent's transfer (conditional on his action), which, in turn, increases the principal's cost of incentivizing the risk-averse agent. In this paper I allow the principal to acquire an additional signal and to choose the signal's precision at a cost. The additional signal allows for a reduction of the risk associated with the transfers, and hence reduce the average transfer to the agent.

This framework posits a trade-off for the principal's monitoring decision between the cost of monitoring and the cost of imposing risk on the agent. Using a simple model with two effort levels, two outcomes, and two signal realizations, I study how the principal's monitoring choice depends on each of two characteristics of the agent: his disutility from performing the task and his probability of succeeding in the task given positive effort.

First, how does the principal's monitoring choice depend on the agent's disutility from performing the task? In a principal-agent problem without monitoring the only instrument the principal can use to maintain the incentive-compatibility constraint that the agent faces is the spread between payoffs. Once monitoring is allowed, the principal is free to mix the two instruments that contribute to holding the incentive-compatibility constraint: increasing the spread or increasing the precision of monitoring. I show that an increase in the disutility from performing the task results in an increase in *both* instruments.

Second, how does the principal's monitoring choice depend on the probability of succeeding in the task? I show that an increase in the probability of succeeding in the task results in a decrease in both the monitoring precision and the spread.

Observed heterogeneity in the agent's characteristics along the two dimensions studied in this paper rises naturally in various principal-agent settings. Consider, for example, the problem of a firm that hires a worker to do some task. This firm may face individuals with different disutility levels of performing the task (e.g., due a to difference in personal circumstances) and different probabilities of succeeding in the task (e.g., due to a difference in experience). In this context the model dictates how the firm that chooses both the worker's payoff and the quality of monitoring should take into account all those observed sources of heterogeneity.[1]

Similarly, these sources of heterogeneity rise in the problem of insuring an agent against some adverse event. The task in this context is being precautious. The insurer may face individuals

[1] Notice that the model includes moral hazard but not adverse selection.

with different disutility levels of performing the task (e.g., due to a difference in attitudes towards being precautious) and different probabilities of succeeding in the task (e.g., due to a difference in probabilities of experiencing the adverse event).

Demougin and Fluet (2001) provide general results for a principal-agent problem with moral hazard, in which both parties are risk neutral and the agent faces a limited liability constraint. Their analysis emphasizes (money) incentives and monitoring as two inputs in the production of effort. The literature on *contingent monitoring systems* introduces monitoring into the principal-agent problem as well, but emphasizes how monitoring should depend on the outcome of the agent's action rather than the agent's characteristics. Kim and Suh (1992) show that under mild conditions the optimal monitoring investment is decreasing in the outcome. Fagart and Sinclair-Desgagné (2007) extend Kim and Suh's result by showing that when the derivative of the inverse utility is convex (concave) the principal prefers monitoring systems whose precision increases (decreases) with respect to the outcome.

2 The model

A risk-neutral principal contracts with a risk-averse agent. The agent's action $a \in \{0, \overline{a}\}, \overline{a} > 0$ is his private information. This action determines output, which is observed and owned by the principal. The output can be either high (H) or low (L) as follows: $p(H|\overline{a}) = \pi, p(H|0) = 0$. Denote the value for the principal from high output by y and normalize the value of low output to 0.[2] The agent's utility is $u(w) - a$, where $u(\cdot)$ is strictly increasing, strictly concave, and continuously differentiable; w is the transfer to the agent; and a is normalized such that it is the agent's utility cost for exerting effort a.

The principal can acquire, upon the output realization, a binary signal $s \in \{G, B\}$ on the agent's action, representing good and bad outcomes, respectively. The good signal outcome can happen only if the agent's action is $a = \overline{a}$, i.e., $p(G|0) = 0$. This means that the accuracy of the signal is determined by $p(G|\overline{a})$. If $p(G|\overline{a}) = 0$, the signal carries no information; if $p(G|\overline{a}) = 1$, the signal perfectly reveals the agent's action. Denote $p(G|\overline{a})$ by θ and let θ be a choice variable of the principal. The signal's cost is a strictly increasing convex function $c(\theta)$.

Since acquiring an informative signal is costly, the principal will always set $\theta = 0$ for an agent with outcome H, as $p(H|0) = 0$ implies that outcome H reveals the agent's action. Thus, there are only three possible outcomes in equilibrium: $\{H, G, B\}$. The contract specifies a recommendation

[2] I assume that y is high enough to justify the costly incentives for the agent.

on action, the precision choice of the monitoring technology when low output is realized, and a transfer to the agent for any outcome. The action recommendation must be incentive compatible. In addition, assume that the agent has an outside option U.

3 The contract

Denote by w^x the principal's transfer to the agent conditional on outcome x for $x \in \{H, G, B\}$. Let $\widehat{C}$ be the expected cost for a principal who recommends the action $\overline{a}$. The principal's problem is as follows:

$$\begin{aligned}
&\widehat{C} = \min_{w^H, w^G, w^B, \theta} \left\{\pi w^H + (1-\pi)\,\theta w^G + (1-\pi)\,(1-\theta)\,w^B + (1-\pi)\,c\,(\theta)\right\} \\
&s.t. \\
&\pi u\left(w^H\right) + (1-\pi)\,\theta u\left(w^G\right) + (1-\pi)\,(1-\theta)\,u\left(w^B\right) - \overline{a} \geq U \\
&\pi u\left(w^H\right) + (1-\pi)\,\theta u\left(w^G\right) + (1-\pi)\,(1-\theta)\,u\left(w^B\right) - \overline{a} \geq u\left(w^B\right) \qquad (1)
\end{aligned}$$

The first constraint is the individual-rationality (IR) constraint. The second constraint is the incentive-compatibility (IC) constraint. The left-hand side of the two constraints is the expected utility for the agent conditional on $a = \overline{a}$. The right-hand side of the IC constraint is the utility for the agent conditional on $a = 0$. Notice that since the IC constraint holds, the objective function assumes the probabilities given action $\overline{a}$.

The following claim determines the ranking of the transfers.

Claim 1 *In the optimal solution* $u(w^H) = u(w^G) > u(w^B) = U$.

All proofs are relegated to the appendix.

Lemmata 2 and 4 show, respectively, that the IR and the IC hold with equality. Use this to rewrite the problem as follows. In the IC, substitute $u(w^B)$ with U and derive $u\left(w^H\right) = U + \frac{\overline{a}}{\pi + (1-\pi)\theta}$. Using the values for $\{w^H, w^G, w^B\}$ in the objective function, leads to the following convex optimization problem, whose solution for θ is identical to that of Problem 1:

$$C = \min_{\theta} \left\{\alpha(\theta) u^{-1}\left(U + \frac{\overline{a}}{\alpha(\theta)}\right) + (1-\alpha(\theta)) u^{-1}\left(U\right) + (1-\pi)\,c\,(\theta)\right\} \qquad (2)$$

where $\alpha \equiv \pi + (1-\pi)\,\theta$.

To understand the role of monitoring precision in this problem, consider the solution to the first best. In the first best, the principal observes the agent's effort, so no monitoring is required. The first-best allocation is then a fixed transfer (independent of output) that is equal to $u^{-1}(U + \overline{a})$. In this case the principal compensates the agent only for his effort.

The principal's cost in Problem 2 differs from the principal's cost in the first best in two aspects. First, in the constrained problem, monitoring may be used upon low output with a cost of $(1-\pi)\,c(\theta)$. Second, in the constrained problem, the principal is required to create a spread in transfers conditional on outcomes. Therefore, the principal delivers to the agent utility as a lottery between $u^{-1}\left(U + \frac{\overline{a}}{\alpha}\right)$ with probability α and $u^{-1}(U)$ with probability $(1-\alpha)$. I refer to the difference between the two utilities $\left(U + \frac{\overline{a}}{\alpha}, U\right)$ as the *spread*.

The average utility delivered through the lottery is $U + \overline{a}$. This is, by construction, equal to the utility delivered in the first best. Therefore, the only role of the signal in this problem is to reduce the risk associated with the spread and thus reduce the cost of delivering utility as a lottery rather than as a certainty equivalent. Indeed, if the signal was without cost, the principal would set $\theta = 1$, and both the allocation and the principal's cost would be identical to those of the first best.

4 Optimal monitoring

Theorem 1 characterizes the contract w.r.t. the disutility from performing the task $\bar{a}$.

Theorem 1 *The solution to Problem 2 has the following characteristics:*
(i) *the optimal signal's precision* (θ) *increases with the task's disutility* $(\bar{a})$*;*
(ii) *the utility spread* $\left(\frac{\overline{a}}{\pi+(1-\pi)\theta}\right)$ *increases with the task's disutility* $(\bar{a})$.

To gain intuition on Theorem 1 consider the principal-agent problem presented here except that monitoring is unavailable. In this environment an increase in the disutility from performing the task compels the principal to increase the spread between payoffs in order to maintain the incentive-compatibility constraint that the agent faces. Once monitoring is allowed the principal is free to mix the two instruments that contribute to holding the incentive-compatibility constraint: increasing the spread or increasing the precision of monitoring. When the disutility from performing the task increases the principal continues to maintain this balance by increasing the marginal cost of both instruments. Notice that since the utility provided to the agent at a low realization of the signal is fixed at U, an increase in the utility spread implies an increase in the payments spread as well.

Theorem 2 deals with the effect of the probability of succeeding in the task (π) on optimal monitoring:

Theorem 2 *The solution to Problem 2 has the following characteristics:*
(i) *the optimal signal's precision* (θ) *decreases with the success probability* (π)*;*
(ii) *the utility spread* $\left(\frac{\overline{a}}{\pi+(1-\pi)\theta}\right)$ *decreases with the success probability* (π).

Without monitoring, an increase in the probability of succeeding in the task allows the principal to induce high effort with a lower utility spread. When monitoring is available the principal decreases both the utility spread and the monitoring precision maintaining the balance between the two instruments.

References

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Online Appendix

Proof of claim 1

Lemma 1 *In the optimal solution either $w^H > w^B$ or $w^G > w^B$, or both.*

Proof. Rewrite the IC as:

$$\pi u\left(w^H\right) + (1-\pi)\,\theta u\left(w^G\right) \geq [\pi + (1-\pi)\,\theta]\, u\left(w^B\right) + \overline{a}. \tag{3}$$

Since $\overline{a} > 0$ and since the sum of the coefficients of $\left\{u\left(w^H\right), u\left(w^G\right)\right\}$ is equal to the coefficient of $u\left(w^B\right)$ (and positive), if both $w^H \leq w^B$ and $w^G \leq w^B$ then the IC cannot hold. ■

Lemma 2 *In the optimal solution the IR holds with equality.*

Proof. The solution with a slack IR can be improved by decreasing w^B by ε. For a small ε, the IR is still slack. The IC remains slack or becomes slack (see (3)). The objective function increases by ε, which is a contradiction to the solution being optimal. ■

Lemma 3 *In the optimal solution $w^H = w^G$.*

Proof. The outcomes H and G have identical informational content. Therefore, an optimal solution that sets $w^H \neq w^G$ presents the risk-averse agent with a lottery that does not depend on any action and therefore does not support any incentive. The contract can be improved by replacing the lottery with the certainty equivalence of that lottery that would cost less to the planner.

Formally, assume that $w^H > w^G$. The optimal solution can be improved as follows. Decrease w^H by ε and increase w^G by $\frac{\pi\varepsilon}{(1-\pi)\theta}$. By construction this change does not affect the objective function because $\pi\left(w^H - \varepsilon\right) + (1-\pi)\,\theta\left(w^G + \frac{\pi}{(1-\pi)\theta}\right) = \pi w^H + (1-\pi)\,\theta w^G$. To study the effect on the IR, consider the part of the IR composed of $\pi u\left(w^H\right) + (1-\pi)\,\theta u\left(w^G\right)$. The change makes the IR slack because it is a lottery with the same certainty equivalent but with less risk. Formally, the claim is that:

$$\begin{gathered}\pi u\left(w^H - \varepsilon\right) + (1-\pi)\,\theta u\left(w^G + \frac{\varepsilon\pi}{(1-\pi)\,\theta}\right) > \pi u\left(w^H\right) + (1-\pi)\,\theta u\left(w^G\right) \\ (1-\pi)\,\theta\left(u\left(w^G + \frac{\varepsilon\pi}{(1-\pi)\,\theta}\right) - u\left(w^G\right)\right) > \pi\left(u\left(w^H\right) - u\left(w^H - \varepsilon\right)\right)\end{gathered} \tag{4}$$

Divide both sides by ε and rearrange to get:

$$\frac{u\left(w^H\right) - u\left(w^H - \varepsilon\right)}{\varepsilon} < \frac{u\left(w^G + \frac{\varepsilon\pi}{(1-\pi)\theta}\right) - u\left(w^G\right)}{\frac{\varepsilon\pi}{(1-\pi)\theta}} \tag{5}$$

In the limit this is $u'\left(w^{H}\right) < u'\left(w^{G}\right)$, which is true by the negation assumption that $w^{H} > w^{G}$. Thus the IR (and similarly the IC) become slack, which is a contradiction to Lemma 2. Therefore it is impossible for w^{H} to be strictly greater than w^{G} in the optimal solution.

The same line of proof eliminates the possibility of $w^{G} > w^{H}$ by showing that increasing w^{H} by ε and decreasing w^{G} by $\frac{\pi\varepsilon}{(1-\pi)\theta}$ violates Lemma 2.

■

Lemma 4 *In the optimal solution the IC holds with equality.*

Proof. By Lemmata 1 and 3 $w^{H} > w^{B}$. If the IC is slack then the objective function can be improved. Decrease w^{H} by ε and increase w^{B} by $\frac{\varepsilon\pi}{(1-\pi)(1-\theta)}$. By construction this change does not affect the objective function. The IC still holds. Consider the part of the IR composed of $\pi u\left(w^{H}\right) + (1-\pi)(1-\theta) u\left(w^{B}\right)$. Those changes make the IR slack because it is a lottery with the same certainty equivalent but with less risk. Formally, the claim is that:

$$\pi u\left(w^{H}-\varepsilon\right) + (1-\pi)(1-\theta) u\left(w^{B} + \frac{\varepsilon\pi}{(1-\pi)(1-\theta)}\right) > \pi u\left(w^{H}\right) + (1-\pi)(1-\theta) u\left(w^{B}\right)$$

$$\pi\left(u\left(w^{H}\right) - u\left(w^{H}-\varepsilon\right)\right) < (1-\pi)(1-\theta)\left(u\left(w^{B} + \frac{\varepsilon\pi}{(1-\pi)(1-\theta)}\right) - u\left(w^{B}\right)\right). \tag{6}$$

Divide both sides by ε and rearrange to get:

$$\frac{u\left(w^{H}\right) - u\left(w^{H}-\varepsilon\right)}{\varepsilon} < \frac{u\left(w^{B} + \frac{\varepsilon\pi}{(1-\pi)(1-\theta)}\right) - u\left(w^{B}\right)}{\frac{\varepsilon\pi}{(1-\pi)(1-\theta)}}. \tag{7}$$

In the limit this is $u'\left(w^{H}\right) < u'\left(w^{B}\right)$, which is true because $w^{B} < w^{H}$. Now decrease w^{H} by δ in order to improve the objective function without damaging any of the constraints. ■

Lemma 5 *In the optimal solution $u\left(w^{B}\right) = U$.*

Proof. Since both the IR and the IC are tight, and since the LHS of both constraints is identical, the RHS of both constraints is equal and $u\left(w^{B}\right) = U$. ■

Claim 1 is then a combination of Lemmata 1, 3, and 5.

Proof of Theorem 1

Theorem 1 *The solution to Problem (2) has the following characteristics:*
(i) the optimal signal's precision (θ) increases with the task's disutility $(\bar{a})$;
(ii) the utility spread $\left(\frac{\bar{a}}{\pi+(1-\pi)\theta}\right)$ increases with the task's disutility $(\bar{a})$.

Proof. The two parts of the theorem are proved sequentially.

Proof of (i)
The proof is based on monotone comparative statics.

$$\begin{aligned}\frac{\partial w}{\partial \overline{a}} &= \left(u^{-1}\right)'\left(U+\frac{\overline{a}}{\alpha}\right) \\ \frac{\partial^2 w}{\partial \overline{a}\partial\theta} &= -\frac{(1-\pi)\,\overline{a}}{\alpha^2}\left(u^{-1}\right)''\left(U+\frac{\overline{a}}{\alpha}\right)<0\end{aligned} \tag{8}$$

According to the monotone comparative statics theorem, θ^* (weakly) increases with $\overline{a}$ if $\frac{\partial^2 w}{\partial \overline{a}\partial\theta} \leq 0$.[3]

Proof of (ii)

Inspection of the spread ($\frac{\overline{a}}{\alpha} = \frac{\overline{a}}{\pi+(1-\pi)\theta}$) shows that the effect of an increase in both $\overline{a}$ and θ on the spread is ambiguous. A further inspection into the FOC shows, however, that this is not the case. The FOC is:

$$c'(\theta) = \frac{\bar{a}}{\alpha}\left(u^{-1}\right)'\left(U+\frac{\overline{a}}{\alpha}\right) - \left(u^{-1}\right)\left(U+\frac{\overline{a}}{\alpha}\right) - \left(u^{-1}\right)(U) \tag{9}$$

As θ increases the LHS increases because $c(\cdot)$ is convex in θ. By differentiating the RHS w.r.t. the spread it can be verified that the RHS is increasing in the spread. Therefore an increase in θ must be accompanied by an increase in the spread. ■

Proof of Theorem 2

Theorem 2 *The solution to Problem (2) has the following characteristics:*
(i) *the optimal signal's precision* (θ) *decreases with the success probability* (π)*;*
(ii) *the utility spread* $\left(\frac{\overline{a}}{\pi+(1-\pi)\theta}\right)$ *decreases with the success probability* (π).

Proof. The two parts of the theorem are proved sequentially.

Proof of (i)
The proof follows the same line of proof as in theorem 1.

$$\begin{aligned}\frac{\partial w}{\partial \pi} &= (1-\theta)\left(\left(u^{-1}\right)\left(U+\frac{\overline{a}}{\alpha}\right)-\frac{\overline{a}}{\alpha}\left(u^{-1}\right)'\left(U+\frac{\overline{a}}{\alpha}\right)-\left(u^{-1}\right)'(U)\right)-c(\theta) \\ \frac{\partial^2 w}{\partial \pi\partial\theta} &= \frac{(1-\pi)\,(1-\theta)\,\overline{a}^2}{\alpha^3}\left(u^{-1}\right)''\left(U+\frac{\overline{a}}{\alpha}\right)>0,\end{aligned} \tag{10}$$

[3]In the case of a maximization problem *supermodularity* is required between $\{\overline{a},\theta\}$. Here the sign of the cross derivative is opposite because it is a minimization problem (as $\min w = -\max -w$).

where the derivation of $\frac{\partial^2 w}{\partial \pi \partial \theta}$ uses the fact that the FOC w.r.t. θ is zero at the optimum. This establishes by the monotone comparative statics theorem that as the parameter π increases, the monitoring precision θ decreases.

Proof of (ii)

Inspection of the spread's denominator $(\pi + (1 - \pi)\theta)$ shows that the effect of a decrease in both π and θ on the spread is ambiguous. However, as directly given by the proof of Theorem 1 (ii) a decrease in θ must be accompanied by a decrease in the spread to keep the FOC.

■